T-61.246 Digital Signal Processing and Filtering

Final / 2nd Mid Term Exam, December 18, 2001 at 16-19.
You are allowed to have a math reference book and a (graphical, programmable) calculator. Storing additional material into the memory of the calculator is strictly forbidden.

Write down on the paper which exam you are taking. Note that if you took the 2nd mid term exam on December 12, you cannot take it again in this event.

Problems:	**2nd mid term exam:**	**3, 4, 5, 6, and 7**
	final exam:	**1, 2, 5, 6, and 7**

1. (6p, FINAL EXAM ONLY) Consider the following flow diagram of a digital system $H(z)$:

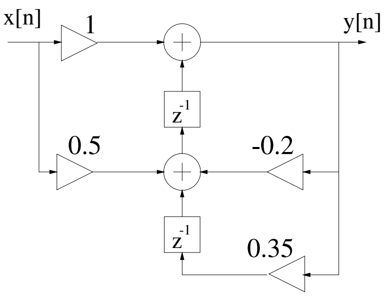

Figure 1: The flow diagram of Problem 1.

   a) Write the corresponding difference equation using x and y.
   b) What is the transfer function $H(z)$ of the filter?
   c) Is the filter stable? Why?
   d) What is the impulse response $h[n]$ of the filter?

2. (6p, FINAL EXAM ONLY) Let us examine the signal

$$x(t) = \sum_{\langle k \rangle} A_k cos(2\pi f_k t + \theta_k)$$

where $k = \{1, 2\}$ so that $A_1 = 2,\ f_1 = 6,\ \theta = 0$, and $A_2 = 0.5,\ f_2 = 12,\ \theta = 0$.

   a) Sketch the signal $x(t)$ in the range $0 \ldots 0.5$ seconds and its spectrum $|X(j\omega)|$ in the range $0 \ldots 20$ Hz.
   b) What is the length of the period of $x(t)$?
   c) Let us sample the signal $x(t)$ with sampling frequency 20 Hz. Draw the spectrum $|X(e^{j\omega})|$ of the obtained discrete signal $x[n]$.
   d) In general, how can the effect of the phenomenon occurred in part c) be reduced?

3. (3p, MID TERM EXAM ONLY) Are the following statements true or false? A right answer gives +0.5 points, no answer 0 points, and a wrong answer -0.5 points. The total point amount for this problem is, however, between 0–3 points.

   a) Structures in which the coefficient values (or their opposite numbers) can be directly found in the flow diagram are called direct form structures.
   b) In a linear-phase filter, we can utilize cofficient symmetry and thus reduce the number of multiplications needed in the implementation.

c) In the impulse-invariant method, the impulse response $h[n]$ of the digital IIR filter can be obtained from the coefficients of the corresponding analog filter $H_a(s)$.

d) A nonlinear mapping (frequency distortion) of the frequency axis happens in bilinear transform. Statement: If the desired cutoff frequency of a digital lowpass filter is very close to one half of the sampling frequency, the significance of the frequency distortion is reduced and, in practical implementations, compensating it by prewarping can be neglected.

e) The numbers of computation steps (general complexities) in the FFT and DFT algorithms are $O(N\,log_2 N)$ and $O(N^2)$, respectively. Statement: When the length of the sequence to be transformed is $N = 128 = 2^7$, the FFT is over 10 times more effective than DFT (when computed with the above general complexities).

f) Truncating a discrete signal (input sequence) with a Hann window $w_{Hn}[n]$ has no effect in the spectrum of the signal.

g) Using CD (compact disc) level 16-bit word lengths, the signal may contain 44100 different quantization levels.

h) In a FIR structure, quantizing signals immediately after multiplication blocks reduces the variance of the total error when compared to only one quantization block in the end of the filter.

4. (3p, MID TERM EXAM ONLY) Consider a second-order IIR filter whose specifications are fulfilled with the following transfer function:

$$H(z) = \frac{1}{1 + a\,z^{-1} + b\,z^{-2}}$$

where $a = -1.40$ and $b = 0.98$. The amplitude response of the filter is shown in the figure below. Let us implement the filter (in direct form) so that the only available values for the coefficients a and b are $\{-\frac{7}{2}, -\frac{6}{2}, -\frac{5}{2}, \cdots, \frac{5}{2}, \frac{6}{2}, \frac{7}{2}\}$. Use rounding and write down the transfer function of the quantized filter and compare its behavior to the original filter.

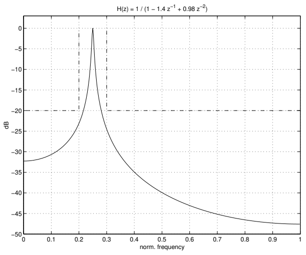

Figure 2: The specifications and the amplitude response of the original filter in Problem 4.

5. (6p, BOTH EXAMS) Consider a filter whose impulse response is $h_0[n] = \delta[n] + 2.5\delta[n-1] + \delta[n-2]$.

a) Draw the zero-pole diagram of the filter $H_0(z)$. What is the type of the filter?

b) Let us define a new filter

$$h_1[n] = \begin{cases} h_0[n/2], & \text{if } n \text{ even} \\ 0, & \text{if } n \text{ odd} \end{cases}$$

Draw the zero-pole diagram of $H_1(z)$. What is the type of this filter?

c) Let us continue once more the definition as in part b) and define $h_2[n]$

$$h_2[n] = \begin{cases} h_1[n/2], & \text{if } n \text{ even} \\ 0, & \text{if } n \text{ odd} \end{cases}$$

Sketch the amplitude response of the filter $H_2(z)$.

6. (6p, BOTH EXAMS)

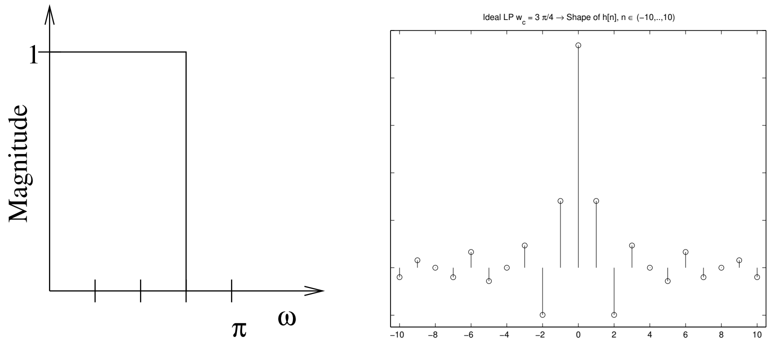

Figure 3: The amplitude response and the shape of the impulse response $h[n]$ (when $n \in (-10 \ldots 10)$) of an ideal lowpass filter $H(e^{j\omega})$, $w_c = 3\pi/4$.

a) Design a FIR-type causal lowpass filter with cutoff frequency $3\pi/4$. Choose 4 as the order of the filter ($M = 2$). Use the truncated Fourier transform method (rectangular window).

b) Design a corresponding filter using the Hann window function $w_h[n]$:

$$w_h[n] = \frac{1}{2}\left[1 + \cos\left(\frac{2\pi n}{2M}\right)\right], \quad -M \le n \le M$$

c) Explain how the frequency responses of the filters designed in parts a) and b) differ assuming that the window size is high enough (e.g. $M = 50$).

7. (6p, BOTH EXAMS) Consider the flow diagram shown in the figure below. It generates sequences $y_1[n]$ and $y_2[n]$. Solve these sequences when $X(z) = A$.

Note that the multiplication coefficients are ordinary constants, which have been parameterized using sine and cosine functions.

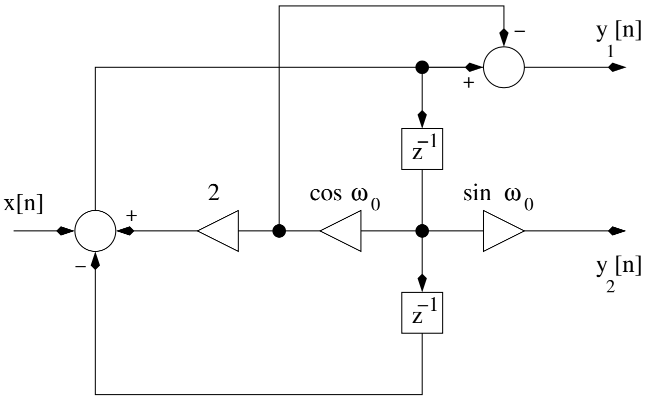

Figure 4: The flow diagram of Problem 7.

Some equations:

$$e^{j\theta} = \cos(\theta) + j\ \sin(\theta)$$

$$y[n] = x[n] \circledast h[n] = h[n] \circledast x[n] = \textstyle\sum_{k=-\infty}^{+\infty} h[k]x[n-k]$$

$$X(e^{j\omega}) = \textstyle\sum_{n=-\infty}^{+\infty} x[n]e^{-j\omega n}$$

$$x[n] = \tfrac{1}{2\pi}\textstyle\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n}\mathrm{d}\omega$$

$$\textstyle\sum_{n=0}^{+\infty} a^n = \dfrac{1}{1-a} \qquad , |a| < 1$$

$$\sin(x)/x \to 1, \text{ when } x \to 0$$

$$\sqrt[N]{c} = \sqrt[N]{re^{j\theta}} = |\sqrt[N]{r}|\, e^{j(\theta+2\pi k)/N} \quad , k = 0, 1, 2, \cdots, N-1$$

$$H(e^{j\omega}) = |H(e^{j\omega})|e^{j\theta(\omega)}, \text{ where } \theta(\omega) = \arg\{H(e^{j\omega})\}$$

$$\tau(\omega) = -\frac{d\theta(\omega)}{d\ \omega}$$

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

$$\mu[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

$$\tfrac{\sin(Wn)}{\pi\, n} = \tfrac{W}{\pi}\mathrm{sinc}(\tfrac{Wn}{\pi}) \leftrightarrow \begin{cases} 1, & 0 \leq |\omega| \leq W \\ 0, & W < |\omega| \leq \pi \end{cases}$$

Z-transform:

$$X(z) = \textstyle\sum_{n=-\infty}^{\infty} x[n]\, z^{-n}$$

$$x[n] \leftrightarrow X(z)$$

$$x[n-n_0] \leftrightarrow z^{-n_0}\, X(z)$$

$$a\, x[n] \leftrightarrow a\, X(z)$$

$$a^n\, \mu[n] \leftrightarrow 1/(1 - a\, z^{-1})$$

$$H(z) = Y(z)/X(z)$$

$$\cos(\theta n)u[n] \leftrightarrow \frac{1-\cos(\theta)z^{-1}}{1-2\cos(\theta)z^{-1}+z^{-2}}, \qquad |z| > 1$$

$$\sin(\theta n)u[n] \leftrightarrow \frac{\sin(\theta)z^{-1}}{1-2\cos(\theta)z^{-1}+z^{-2}}, \qquad |z| > 1$$

Bilinear transform:

$$s = (2/T)(1 - z^{-1})/(1 + z^{-1})$$

$$\Omega_c = (2/T)\ \tan(\omega_c T/2),$$

where Ω and ω are the angular frequencies of the analog and discrete filters, respectively