T-61.246 Digital Signal Processing and Filtering

Exam / 2nd mid term exam 13th Dec 2002 at 12-15. Hall M.
You cannot attend to 2nd MTE, if you were in the MTE on 11th Dec 2002.
If you are doing the 2nd MTE, reply to problems 3, 5, 6, 7.
If you are doing the exam, reply to problems 1, 2, 4, 6, 7.
Write down in your paper, if you are doing 2nd MTE or exam.

You may use a (graphical) calculator. You must clear all extra memory in your calculator.
There is an additional formulae table given in the exam.

1. (Exam, 6p) Consider a linear, time-invariant, stable and causal discrete system, where the input $x[n]$ and output $y[n]$ are:

n	$x[n]$	$y[n]$
-1	0	0
0	1	2
1	2	1
2	-1	?
3	0	?
4	1	?
5	0	?

   a) Define the impulse response $h[n]$ of the system using $x[n]$ and $y[n]$, and conditions that initial values of system are zero and it is form (a, b, c and d constants):

$$h[n] = \begin{cases} a, & \text{when } n < 0 \\ b, & \text{when } n = 0 \\ c, & \text{when } n = 1 \\ d, & \text{when } n > 1 \end{cases}$$

   b) Calculate the missing values of $y[n]$.

2. (Exam, 6p) Consider a filter whose difference equation is
$y[n] = x[n] - 0.8x[n-1] - 1.6y[n-1] - 0.68y[n-2]$.

   a) Draw the block (flow) diagram of the filter.

   b) What is the transfer function $H(z)$ of the filter?

   c) Draw the pole-zero-diagram of the filter. Sketch the amplitude response $|H(e^{j\omega})|$. Is the filter low/high/bandpass/bandstop?

   d) Compute the first values of the impulse response h[n] when $n = 0 \ldots 3$. The delay registers are initialized to zero. The closed form of $h[n]$ is not required.

   e) Is the filter stable?

3. (2nd MTE, 6p) Are the following statements true (T) or false (F)? A right answer gives +1 point, no answer 0 points, and a wrong answer –1 point. Reply to as many statements as you want; no explanations are needed. The total point amount for this problem is, however, between 0-6 points. If you want explicitely to comment on your choices, write down them separately.

1) The coefficients of the transfer function of the filter can be directly (or negative) seen in the direct form block diagram.

2) The transfer function $H(z) = h[0](1 - z^{-4}) + h[1](z^{-1} - z^{-3})$ corresponds a linear-phase FIR filter.

3) One possible polyphase realization for the FIR filter
$H(z) = 1 + 0.3z^{-1} - 0.3z^{-2} - z^{-3}$ is $H(z) = E_0(z^2) + E_1(z^2)$, where
$E_0(z) = 1 - 0.3z^{-1}$ and $E_1(z) = 0.3 - z^{-1}$.

4) The order of digital FIR filter is typically bigger than that of a digital IIR filter with corresponding specifications.

5) A transfer function of a highpass filter is given by
$H(z) = K(-0.1219 - 1.3992z^{-1} + 7.0422z^{-2} - 1.3992z^{-3} - 0.1219z^{-4})$.
Statement: The coefficient K has to be $K = 0.25$, so that the maximum of the filter is scaled to unity (one).

6) In the bilinear transform the whole frequency band $(0 \ldots \infty)$ of an analog filter is mapped to the possible frequency band $(0 \ldots f_s/2)$. of an digital IIR filter.

7) The truncation of a discrete signal (sequence) using, for example, rectangular window causes distortion in the spectrum of the signal.

8) The oscillatory behaviour at the edge frequencies, known as Gibbs phenomenon, can be effectively decreased by using a Hamming window $w_{hamm}[n]$, but at the same the frequency selectivity is getting worse (transition from passband to stopband is slower).

9) Sign bit $s = 1$ corresponds a negative number. Statement: The two's complement of a decimal number -0.375 using a sign bit and four bits is represented by $1_\Delta 101$.

10) The scaling of signal in order to supress the overflows in a filter also gives a better signal-to-noise ratio (SNR) in the filter.

11) The sampling frequency (multirate) is decreased by a factor $M = 3$,
$f_s = (1/3)f_{s,old}$. Statement: The cut-off frequency of the lowpass filter used in decimation (decimation filter) can be at most $\pi/3$ (ideal filter) in order not to cause aliasing in the signal.

12) The sampling frequency (multirate) is increased by a factor L, $f_s = Lf_{s,old}$. Statement: There will be L additional spectra (image) in the baseband.

4. (Exam, 6p) The one-side spectrum $X(j\omega)$ of a continuous-time periodic signal $x(t)$ is shown in Figure 1. Suppose that all components are in the same phase (zero). The signal is of form $x(t) = \sum_{k=1}^{3} A_k \cos(2\pi f_k t)$

a) What is the fundamental period $T_0 = 1/f_0$ of the signal $x(t)$?

b) Sample the signal using the sampling frequency $f_s = 10$ kHz. Sketch the discrete-time spectrum $X(e^{j\omega})$.

c) The original signal in Figure 1 is first filtered using so called anti-aliasing filter

$$H(j\omega) = \begin{cases} 1, & |f| < 5 \text{ kHz} \\ 0, & |f| > 6 \text{ kHz} \end{cases}$$

After that the signal is sampled again with $f_s = 10$ kHz. Draw not the spectrum $X(e^{j\omega})$.

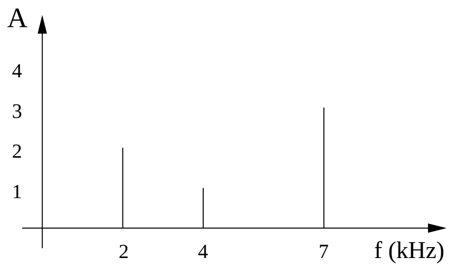

Figure 1: Spectrum of the Problem 4.

5. (2nd MTE, 6p) Consider a filter in Figure 2.

a) What is the transfer function $H(z)$.

b) Draw the direct form canonic structure of the filter.

c) Draw the pole-zero-diagram. Compute the lengths of zeros and poles from the origin. Sketch the amplitude response $|H(e^{j\omega})|$. What type of filter is it (notice one kind of symmetry of coefficients)?

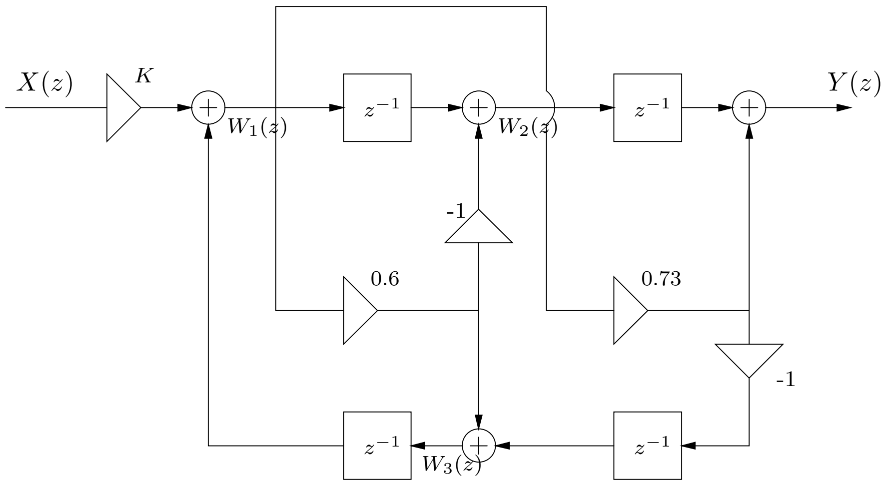

Figure 2: The filter of the problem 5.

6. (2nd MTE and Exam, 6p) Consider an analog filter whose s-plane transfer function and impulse response are $H_a = 1/(s+1) \quad \leftrightarrow \quad h_a(t) = e^{-t}\mu(t)$.

   a) Determine the transfer function $H_I(z)$ of the digital filter corresponding the analog filter $H_a(s)$, when impulse-invariant method is used in the filter digitalization. In other words, take samples from the impulse response $h_a(t)$ of the analog filter at moments $t = nT_s$, where T_s is the sampling period.

   b) Determine the transfer function $H_B(z)$ of the digital filter corresponding the analog filter $H_a(s)$, when bilinear transform is used in the filter digitalization. It is assumed, that the frequency distortions have been taken into account in the design phase of the filter, and they do not need to be compensated. Use $s = (2/T_s)(1 - z^{-1})/(1 + z^{-1})$ for the transform.

   c) Normalize the sampling period to unity, i.e., $T_s = 1$. Compute the zeros and poles of $H_I(z)$ and $H_B(z)$, draw the pole-zero-diagrams, and sketch the amplitude responses of the filters. How do the two methods differ (in general)? Why?

7. (2nd MTE and Exam, 6p) In the figure below there are two block diagrams which both are second order IIR filters. Consider only complex poles of the filters, when real coefficients a and b are quantized so that all possible values are $\{-\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}\}$.
   Draw the all possible positions of complex poles of the two filters and compare them. The transfer function of the filter in right is $H(z) = -bz^{-2}/(1 - 2az^{-1} + (a^2 + b^2)z^{-2})$. Examine the situations when using a small number of bits a narrow LP or a narrow BP filter (bandpass at $0.25f_s$) is designed.

   Notice that the complex poles of a real-coefficient filter are complex conjugates ($p_1 = re^{j\theta}$, $p_2 = p_1^* = re^{-j\theta}$), and the polynomial can be expressed in product form using its roots $1 + d_1z^{-1} + d_2z^{-2} = (1 - p_1z^{-1})(1 - p_2z^{-1})$.