T-61.246 Digital Signal Processing and Filtering

Summer exam, Mon 20.6.2005 12-15, main building.
You are NOT ALLOWED to use any math reference book. **(Graphical) calculator allowed, if extra memory is erased.** Formulas on accompaning paper. **Write down clear intermediate steps. Begin** a new problem **from a new page.**

1) (6p) Multiple choises. Write down a table similar to that one below. Reply one option **A**, **B** or **C**, which is correct or closest. Right answer +0.5 p, wrong answer or no answer 0 p. No explanations needed.

m1	m2	m3	m4	m5	m6	m7	m8	m9	m10	m11	m12

m1) Notation $h[n]$ means normally [**A**] input sequence [**B**] impulse response of the filter [**C**] output sequence.

m2) Convolution is [**A**] multiplication (with respect to time) of two signals, analog or digital, [**B**] addition of two signals, [**C**] basic operation in signal processing, with which it is possible to get output of LTI-system, when input and impulse response are known.

m3) Consider signal which can be expressed $x(t) = A_1 \cos(2\pi f_1 \cdot t + v_1) + A_2 \cos(2\pi f_2 \cdot t + v_2)$. It is filtered with an LTI filter. Constants: $A_i \neq B_i$, $f_i \neq g_i$, $v_i \neq w_i$. Which of the following can be output signal: [**A**] $y(t) = A_1 \cos(2\pi f_1 \cdot t + v_1) + B_2 \cos(2\pi g_2 \cdot t + w_2)$, [**B**] $y(t) = B_1 \cos(2\pi g_1 \cdot t + w_1) + B_2 \cos(2\pi g_2 \cdot t + w_2)$ [**C**] $y(t) = A_1 \cos(2\pi f_1 \cdot t + w_1) + A_2 \cos(2\pi f_2 \cdot t + w_2)$.

m4) LTI filter, whose impulse response is $\{\underline{1}, -1, 1, -1, 1, -1, \ldots\}$ (notation $\underline{a}$ represents the sample at origo) [**A**] is stable, [**B**] is FIR filter, [**C**] has feedback.

m5) The impulse response of a causal second order FIR filter is of form [**A**] $h[n] = a\delta[n+1]+b\delta[n]+c\delta[n-1]$, [**B**] $h[n] = a\delta[n]+b\delta[n-1]+c\delta[n-2]$, [**C**] $h[n] = a\delta[n]+b\delta[n-1]$, where a, b and c are non-zero constants.

m6) What is the number of poles outside origo of the filter $y[n] - 0.5y[n-1] = x[n] + 0.33[n-1] - 0.44x[n-2]$? [**A**] 1 [**B**] 2 [**C**] 3.

m7) What can you say about the filter $H(z) = [0.2 - 0.5z^{-1} + z^{-2}]/[1 - 0.5z^{-1} + 0.2z^{-2}]$? [**A**] Filter is all-pass [**B**] Filter is FIR [**C**] The phase response is linear.

m8) The signal $x(t)$ is sampled with sampling frequency f_s, and the length of the sequence $x[n]$ becomes 80000. If the sampling period T_s was doubled, what would be the length of the sequence $x[n]$? [**A**] 40000, [**B**] 80000, [**C**] 160000.

m9) The ripple in amplitude response known as Gibb's phenomen can be removed by [**A**] increasing the order of the filter [**B**] using e.g. Hamming window [**C**] taking the absolute value from the frequency response.

m10) Minimum-phase filter: [**A**] all poles are outside unit circle [**B**] all zeros are inside unit circle [**C**] all zeros are in origo.

m11) The sampling frequency of the sequence is decreased by a factor (3/5). There are proper anti-alias and anti-imaging filters $H_i(z)$ availabe. Which of the following works? Next page [**A**] Figure 1(a) [**B**] Figure 1(b) [**C**] Figure 1(c).

m12) Sequence $x[n]$ of Problem 2 is fed into the multirate system shown in Figure 1(d). The output $y[n]$ is [**A**] $\{\underline{2.127}, 0, -1.314, 0, \ldots\}$
[**B**] $\{\underline{2.127}, -1.314, 0.000, 1.314, \ldots\}$ [**C**] $\{\underline{2.127}, 0.191, -1.314, -1.309, \ldots\}$.

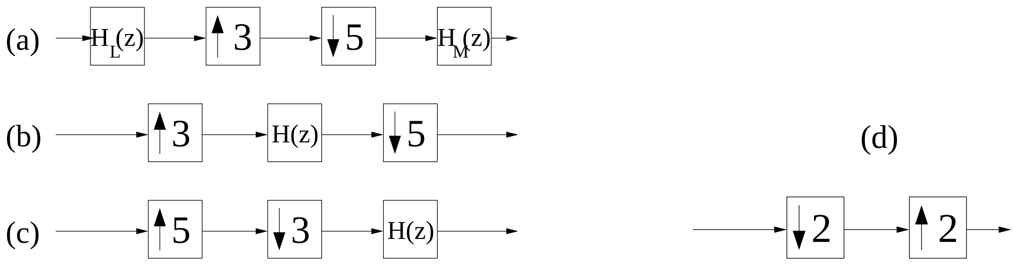

Figure 1: Figures for Problem 1.

2) (6p) Input sequence $x[n]$ is fed into second order causal FIR filter (with empty registers), and the result is output $y[n]$. The first values of sequences are drawn in Figure 2 and the values are:

$$
\begin{aligned}
x[n] &= \{\underline{2.127}, 0.191, -1.314, -1.309, 0.000, 1.309, 1.314, -0.191, -2.127, -3, \ldots\} \\
y[n] &= \{\underline{2.127}, -4.063, 0.431, 1.510, 1.304, 0.000, -1.304, -1.510, -0.431, 1.063, \ldots\}
\end{aligned}
$$

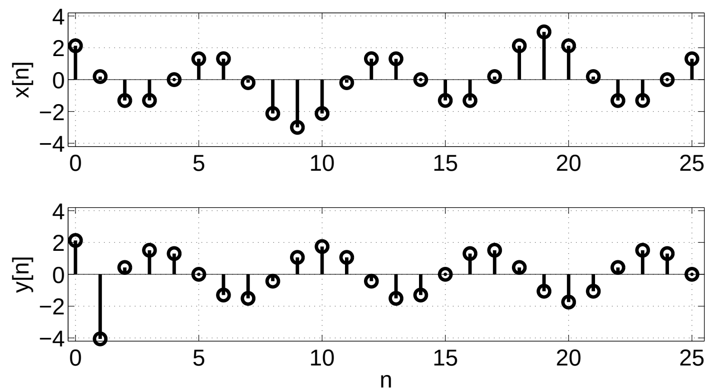

Figure 2: Problem 2, input $x[n]$ and output $y[n]$.

a) What is the impulse response of the filter $h[n]$?
b) Draw the flow (block) diagram of the filter.

3) (6p) There are two poles in origo and two zeros at $z = -1$.

a) Draw the pole-zero plot of the filter and determine and sketch the amplitude response of the filter. Is the filter lowpass / highpass / bandpass / bandstop / all-pass?
b) Filter can be expressed using poles p_i and zeros z_i

$$H(z) = K \cdot \frac{(1 - z_1 z^{-1}) \cdot (1 - z_2 z^{-1}) \cdot \ldots \cdot (1 - z_M z^{-1})}{(1 - p_1 z^{-1}) \cdot (1 - p_2 z^{-1}) \cdot \ldots \cdot (1 - p_N z^{-1})}$$

where K is scaling factor. Write down the transfer function of the filter in the form

$$H(z) = \frac{b_0 + b_1 z^{-1} + \ldots + b_M z^{-M}}{a_0 + a_1 z^{-1} + \ldots + a_N z^{-N}}$$

so that the maximum of the amplitude response is scaled to unity. What is the order of the filter?
c) What is the difference equation of the filter?

4) (6p) **Reply to EITHER 4A OR 4B.**

4A) Design a FIR filter with window method, when the cut-off frequency of the lowpass filter is at $f_c = 2000$ Hz and the sampling frequency is $f_T = 10000$ Hz. Window functions are represented in Table 1.

a) Sketch the frequency response of the ideal $H_{ideal}(f)$.

b) Compute the impulse response $h_{ideal}[n]$. of the corresponding ideal filter. Give the values, when $n = -2 \ldots 2$.

c) Compute the coefficients of the FIR filter $h_{FIR}[n]$ using window method and Hamming window $w_H[n]$, whose length is 5 ($M = 2$).

d) Examine the usefulness of this FIR filter, when in stopband 54.5 decibel minimum attenuation is required.

Window	$w[n]$, $-M \leq n \leq M$	Length of main lobe Δ_{ML}	Relative side lobe A_{sl}	Mininum stopband attenu-ation	Length of transition band $\Delta\omega$
Rectangular	1	$4\pi/(2M+1)$	13.3 dB	20.9 dB	$0.92\pi/M$
Hann	$0.5 + 0.5\cos(\frac{2\pi n}{2M})$	$8\pi/(2M+1)$	31.5 dB	43.9 dB	$3.11\pi/M$
Hamming	$0.54 + 0.46\cos(\frac{2\pi n}{2M})$	$8\pi/(2M+1)$	42.7 dB	54.5 dB	$3.32\pi/M$
Blackman	$0.42 + 0.5\cos(\frac{2\pi n}{2M}) + 0.08\cos(\frac{4\pi n}{2M})$	$12\pi/(2M+1)$	58.1 dB	75.3 dB	$5.56\pi/M$

Table 1: Properties of window functions.

4B) Essay: Digital linear and time-invariant FIR and IIR filters: similarities and differences of filter types, and about basic filter design methods.

5) (6p) **Reply to EITHER 5A OR 5B OR 5C.**

5A) See the filter in Figure 3. The input values are represented with B bits. After multiplications the number of bits is $2B$. In order to get the number of bits in output to B, it is necessary to quantize values of $w[n]$ (block Q).

Quantization error can be compensated using so called error feedback (or error-shaping filter). In Figure 3 there is a second order filter with a second order error feedback system.

Write down first the difference equations for $e[n]$ and $w[n]$, and write down then in frequency domain the quantized output $Y(z)$ using input $X(z)$ and quantization noise $E(z)$, and reply

a) how does the filter behave, if it is possible to use infinite wordlength, i.e. there is no quantization and $e[n] \equiv 0, \forall n$?

b) how does the spectrum of the total noise $E_{tot}(z)$ look like if there is no compensation, i.e. $k = 0$, and if $e[n]$ is white noise so that $E(z) = 1$ for all frequencies?

c) with which simple value of k the effect of noise is suppressed in the passband?

5B) Essay: What do you know about human speech / voice? Compare different tools (Matlab, Tcl/Snack, TI C6711 DSP-kit) using the experience gained in the summer course.

5C) Essay: FFT-algorithms, especially "Decimation-in-Time" and "Decimation-in-Frequency". You do not have to derive formulas.

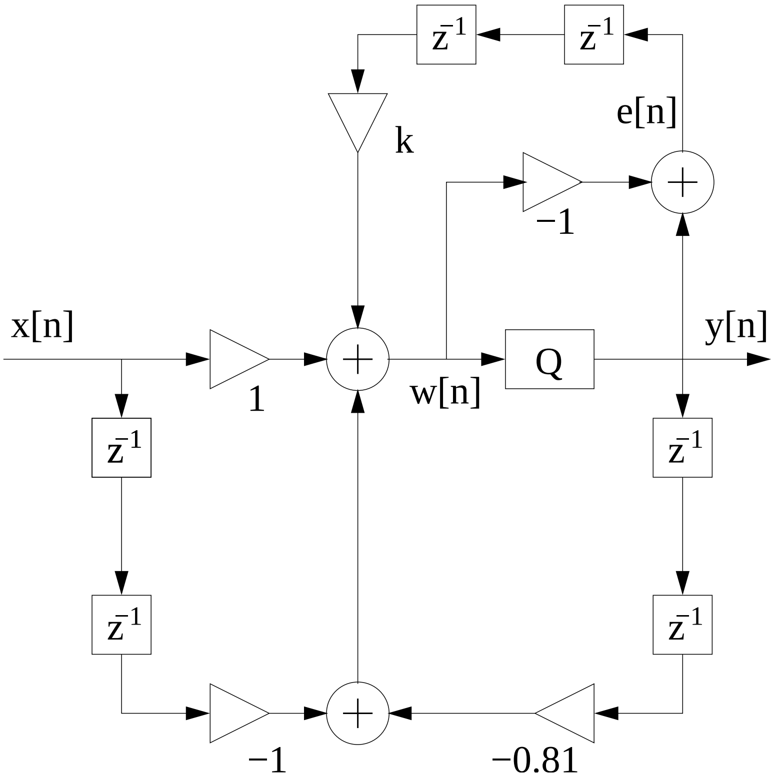

Figure 3: Second order system with second order error feedback.

Have a nice Summer! Deadline for the project work 31.8.2005.