Tik-61.145 Principles of digital signal processing

Exam/additional partial exam, May 21, 1996

1st partial exam consists of problems 1, 2, 3 and 4.

2nd partial exam consists of problems 5, 6, 7 and 8.

Exam consists of problems 1, 4, 5, 7 and 8.

1. (6p) What are the outputs $y_1(n)$ and $y_2(n)$ of the system in Figure 1 at moment $n > 0$, when at $n = 0$ the upper register is preset to value A, and lower to 0 (*hint*: add the system an input branch, solve the transfer functions, and study the scaled impulse response)?

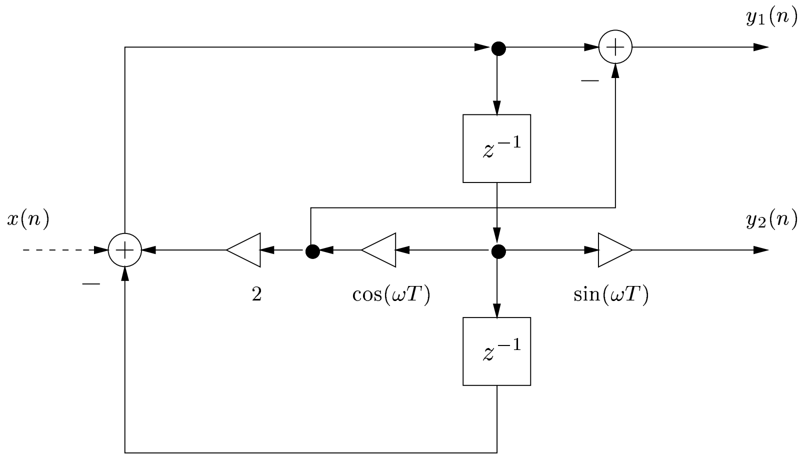

Figure 1: Flow diagram.

2. (6p) Consider the following two systems with impulse responses

$$\begin{aligned} h_1(n) &= \delta(n) + 2\delta(n-2) + \delta(n-4) \\ h_2(n) &= \delta(n) - \delta(n-4). \end{aligned}$$

   (a) Determine the impulse response and the transfer function of the composite cascade of the two systems.

   (b) Determine the amplitude response and the phase response of the composite cascade and sketch the corresponding figures.

   (c) How does the phase of the parallel composite of these two systems behave?

3. (6p) Consider a causal linear transfer invariant system having the transfer function

$$H(z) = \frac{a - z^{-1}}{1 - az^{-1}}, \quad 0 < a < 1, \quad a \in \Re.$$

   (a) Prove analytically that the system is an all-pass system, i.e. the amplitude response of the system is unity for all frequencies.

   (b) Sketch the pole-zero -diagram in the z-plane and shade the area of convergence.

   (c) Prove graphically using the pole-zero -diagram that the amplitude response of the system is constant.

(d) What is the reason for the difference between the analytical result (= 1) and graphical evaluation (=constant)?

4. (6p)

(a) Explain briefly, are the following two systems linear, transfer invariant and/or causal. The systems have been defined by either a difference equation (input/output -relation between input sequence $x(n)$ and output sequence $y(n)$) or the impulse response $h(n)$. $\delta(n)$ is the unit impulse.

(1) $y(n) = a^{x(n)}$

(2) $h(n) = a^n \delta(n-a)$

(b) Is the sequence $x(n) = A\cos\left(\frac{3\pi}{7}n - \frac{\pi}{8}\right)$ periodical? If it is, what is the period?

5. (6p) Consider an FIR filter having the transfer function

$$H(z) = 1 - 2.5z^{-1} + z^{-2}.$$

(a) Draw the pole-zero diagram, and sketch $|H(e^{j\omega T})|$, the magnitude response of the filter. What kind of filter is this?

(b) Draw the pole-zero diagram, and sketch the magnitude response of the same filter, when each delay of the original filter is replaced by four delays. What kind of filter we now have?

6. (6p) Consider a first order recursive causal filter, which has transfer function

$$H(z) = \frac{1}{1 - az^{-1}},$$

where $a = 0.5$.

(a) Is the filter stable?

(b) Study the impulse response of the filter with four bit quantization (sign bit + 3 bits) with rounding arithmetic. In this case, three least significant bits of multiplication result are ignored, and if the first ignored bit was 1, 1 is added to the least significant bit of the four-bit result. It is assumed that the values have been scaled so that their magnitudes are less than one. For example, $\delta(0) = 0.111_2 = \frac{7}{8}$ (greatest positive value) and $a = 0.5 = 0.100_2$. Is the filter now stable? Motivate your answer.

7. Sketch roughly the magnitude responses of Chebysev I and elliptic type IIR digital filters on the interval $[0 \dots \pi]$, when the following specifications have been given. It is assumed that the filters are designed in the continuous domain and they have been digitized using bilinear transform.

   (a) 3rd order lowpass filter, whose
      * passband ends at frequency $\frac{\pi}{3}$ and
      * stopband begins at frequency $\frac{2\pi}{3}$.

   (b) 2nd order highpass filter, whose
      * stopband ends at frequency $\frac{\pi}{4}$ and
      * passband begins at frequency $\frac{\pi}{2}$.

   (c) 10th order bandstop filter, whose
      * 1st passband ends at frequency $\frac{\pi}{4}$,
      * stopband begins at frequency $\frac{\pi}{3}$,
      * stopband ends at frequency $\frac{2\pi}{3}$ and
      * 2nd passband begins at frequency $\frac{3\pi}{4}$.

8. (6p) Consider an analog filter whose s-plane transfer function is

$$H_c(s) = \frac{1}{s+1}.$$

   (a) Determine $H(z)$, the transfer function of the corresponding digital filter, when impulse-invariant transform is used in filter digitalization. (The impulse response of the continuous-time filter is $h_c(t) = e^{-t}$, when $t \geq 0$, 0 otherwise.)

   (b) Determine $H(z)$, the transfer function of the corresponding digital filter, when bilinear transform is used in filter digitalization (transformation formula: $s = \frac{2}{T} \cdot \frac{1 - z^{-1}}{1 + z^{-1}}$). It is assumed, that the frequency distortions have been taken into account in the design phase of the filter, and they do not need to be compensated.

   (c) Sketch the magnitude responses of both digital filters obtained using methods above when $T = 1$. How do the two methods differ (in general)?