T-61.3010 Digital Signal Processing and Filtering

Mid term exam 2 / Final exam. Wed 6.5.2009 9-12. Hall M.

You can do MTE2 only once either 6.5. or 13.5. Mid term exam: Problems 1 and 2.

You can do final exam only once either 6.5. or 13.5. Final exam: Problems 3, 4, 5, 6, and 7. Begin each problem from a new page.

You can have a function calculator. If not, apply variables like $k = \sin(0.2\pi)$. You are not allowed to have any math formula book of your own. You will be given a course formula paper.

1) (12 x 1p, 0-12 p, **ONLY MTE2**) Multichoice. There are 1–4 correct answers, but choose **one and only one.** Fill in **into a separate form**, which will be read optically.

Correct answer +1 p, incorrect −0.5 p, no answer 0 p. You do not need to explain your choices. Reply to as many statements as you want. The maximum points of this problem is 12 and the minimum 0.

1.1 Consider a block diagram in Figure 1, where there are three LTI systems $S(z)$, $R(z)$, and $T(z)$. The tranfer function of the system is

(A) $H(z) = (R(z) + T(z)) \cdot S(z)$
(B) $H(z) = \frac{S(z)+R(z)T(z)}{1-T(z)}$
(C) $H(z) = \frac{S(z)R(z)}{1-S(z)T(z)}$
(D) $H(z) = \frac{R(z)S(z)}{T(z)}$

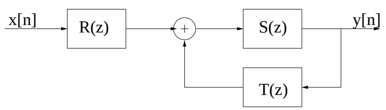

Figure 1: Multichoice 1.1: block diagram.

1.2 Causal and stable LTI filter

$$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{1 - 1.3z^{-1} + 0.4z^{-2}}$$

is depicted in a canonic (with respect to delays) direct form II in

(A) Figure 2(a).
(B) Figure 2(b).
(C) Figure 2(c).
(D) Figure 2(d).

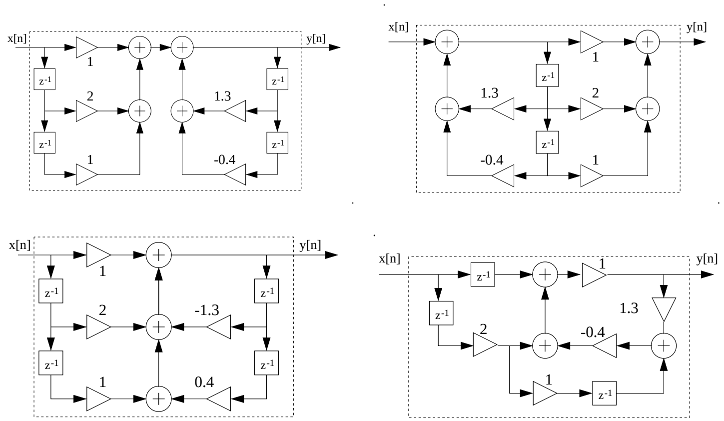

Figure 2: Multichoice 1.2: structures, top row **(A)** and **(B)**, bottom row **(C)** and **(D)**.

1.3 Digital LTI system, with input $x[n]$ and output $y[n]$, is expressed with difference equation pair

$$\begin{aligned} w[n] &= 0.5x[n] + 0.8w[n-1] \\ y[n] &= 0.25x[n-1] - 0.5w[n-1] \end{aligned}$$

(A) Frequency response of the filter is $H(e^{j\omega}) = -0.2 \cdot (e^{-j2\omega})/(1 - 0.8e^{-j\omega})$.
(B) Phase response of the filter is linear.
(C) There is no feedback, so the system is FIR.
(D) There is a feedback without delay ("delay free loop"), and therefore it cannot be realized.

1.4 Bilinear transform

(A) is a bijection (one-to-one mapping), where left half plane of analog s-plane is mapped to inside unit circle in z-plane.
(B) is a mapping, where the frequency axis $j\Omega$ of s-plane is mapped to y-axis of z-plane.
(C) is a transform with which it is possible to shift quantization noise into a stopband of the filter.
(D) is one way to compute analog FIR-filter from corresponding digital filter.

1.5 The oscillation in the passband of a FIR filter is defined in linear axis so that amplitude response may vary between 0.9 and 1.1, see Figure 3, left y-axis.

If the maximum value of the amplitude response is first scaled to 1, which corresponds 0 dB in logarithmic decibel scale, then what is the maximum power (square) oscillation in passband? In other words, what is the value for α_{max} in Figure 3, right y-axis? Hint: Formulae "Logarithm".

(A) $\alpha_{max} \approx 0.0458$ dB
(B) $\alpha_{max} \approx 1.74$ dB
(C) $\alpha_{max} \approx 3.01$ dB
(D) $\alpha_{max} \approx 14.0$ dB

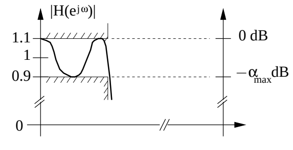

Figure 3: Multichoice 1.5: filter specifications.

1.6 Design a fourth-order digital FIR lowpass filter with window method ("Truncated Fourier series") using Hamming window $w[n]$ (see Formulae). When the cut-off frequency is chosen to be $\omega_c = 0.8\pi$, then the ideal impulse response is $h_{\text{ideal}}[n] \approx \{\ldots, -0.15, 0.19, \underline{0.80}, 0.19, -0.15, \ldots\}$.

What is the final impulse response $h[n]$ of the FIR filter with given precision?

(A) $h[n] = -0.15\delta[n] + 0.19\delta[n-1] + 0.80\delta[n-2] + 0.19\delta[n-3] - 0.15\delta[n-4]$
(B) $h[n] = -0.01\delta[n] + 0.10\delta[n-1] + \delta[n-2] + 0.10\delta[n-3]$
(C) $h[n] = -0.01\delta[n+2] + 0.10\delta[n+1] + 0.80\delta[n] + 0.10\delta[n-1] - 0.01\delta[n-2]$
(D) $h[n] = 0.08\delta[n+2] + 0.54\delta[n+1] + \delta[n] + 0.54\delta[n-1] + 0.08\delta[n-2]$

1.7 The transfer function of a (monotonic) highpass filter is $H(z) = K \cdot (1 - z^{-1})$. If the maximum of the filter is scaled to one, the coefficient K must be

(A) $K = 1/\infty$
(B) $K = 0.5$
(C) $K = 1$
(D) $K = 2$

1.8 Fast Fourier Transform (FFT)

(A) requires some $N^2 \log_2 N^2$ complex operations with large N.
(B) computes discrete Fourier transform (DFT) more accurate if there are more memory and computing power available.
(C) computes exact discrete Fourier transform (DFT) with less operations than that with definition of DFT
(D) was developed in early 1980's in the USA for the need of fast computation in the "Star Wars" missile defence.

1.9 In Matlab you can compute a sum S of seven numbers by giving an expression

`S = 1 - 0.3 - 0.1 - 0.1 - 0.2 - 0.2 - 0.1`

which returns a value `-2.7756e-17` $\approx -2.8 \cdot 10^{-17}$.

(A) Matlab gives for the exactly same sum sequence S randomly different results because Matlab utilises non-deterministic quantum computation.
(B) Matlab applies in the number representation by default 8 bits' accuracy, which leads to "rounding errors".
(C) Matlab example proves that the mathematical sum $S = 1 - 0.3 - 0.1 - 0.1 - 0.2 - 0.2 - 0.1 \neq 0$.
(D) None of above is true.

1.10 The sampling frequency of a digital sequence is to be increased to (3/2) of the original. When having proper decimation filter $H_D(z)$ ("anti-alias") and interpolation filter $H_I(z)$ ("anti-imaging"), what is the correct way to do this?

(A) $x[n] \rightarrow \boxed{H_D(z)} \rightarrow \boxed{\downarrow 2} \rightarrow \boxed{\uparrow 3} \rightarrow y[n]$

(B) $x[n] \rightarrow \boxed{\uparrow 3} \rightarrow \boxed{\downarrow 2} \rightarrow \boxed{H_I(z)} \rightarrow y[n]$

(C) $x[n] \rightarrow \boxed{\uparrow 3} \rightarrow \boxed{H_I(z) \cdot H_D(z)} \rightarrow \boxed{\downarrow 2} \rightarrow y[n]$

(D) $x[n] \rightarrow \boxed{\downarrow 2} \rightarrow \boxed{H_D(z) \cdot H_I(z)} \rightarrow \boxed{\uparrow 3} \rightarrow y[n]$

1.11 The spectrum $|X(e^{j\omega})|$ of a digital real-valued sequence $x[n]$ is given in Figure 4. The sequence is filtered first with lowpass filter, whose cut-off is $\omega_c = \pi/2$, after this downsampled by $M = 2$, highpass filtered ($\omega_c = \pi/2$) and finally downsampled by $M = 2$:

$x[n] \rightarrow \boxed{\text{LP}} \rightarrow \boxed{\downarrow 2} \rightarrow \boxed{\text{HP}} \rightarrow \boxed{\downarrow 2} \rightarrow y[n]$

(A) There is a peak in the spectrum $|Y(e^{j\omega})|$ at $\omega = 0.8\pi$ due to the peak in $|X(e^{j\omega})|$ at $\omega = 0.3\pi$.

(B) In the spectrum $|Y(e^{j\omega})|$ in range $0 \ldots \pi$ there are frequency components of $|X(e^{j\omega})|$ in range $\pi/2 \ldots \pi$.

(C) Connection between sequences $y[n]$ and $x[n]$ is $y[n] = 0.5 \cdot (x[4n] - x[4n-1])$.

(D) If lowpass and highpass filters are considered ideal, then from $y[n]$ it is possible with proper methods to recover the original $x[n]$.

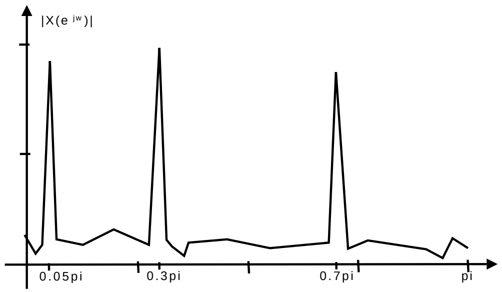

Figure 4: Multichoice 1.11: spectrum $|X(e^{j\omega})|$.

1.12 Read in into Matlab an audio sample $x[n]$ from file `sample.wav`, where the sampling frequency f_T is 8000 Hz. What for the following lines of code are suited best?

```
[x, fT] = wavread('sample.wav');
N       = 205;
thr     = 0.05;  % fix this!

for k = (1 : N : length(x)-N)
  tmpx = x(k : k+N-1);
  mag  = abs(fft(tmpx, N))/N;
  if ((mag(23) > thr) && (mag(39) > thr))
    disp(['Index k = ' num2str(k)])
  end;
end;
```

(A) To compute the total signal energy in small time frames.

(B) To draw the spectrogram of a signal in sequences of 205 numbers.

(C) To recognise those time frames where sinusoidal frequencies 852 Hz and 1477 Hz are powerful.

(D) To lowpass filter two frequency components at each time frame using the "overlap-add" method.

2) (6p, **MID TERM EXAM**) Choose either 2A or 2B. Write down an exam essay, where introduction, body of essay, and summary. Use clear visualisations if needed.

2A) **OPTION A.** FFT algorithms. In addition to common text you can use as an example "radix-2 DIT FFT" algorithm whose butterfly equations and W_N are given in the formula table. Compute with intermediate steps FFT for a sequence ($N = 4$) $x[n] = 3\delta[n] - 1\delta[n-1] - 4\delta[n-2] + 2\delta[n-3]$.

2B) **OPTION B.** Automatic speech recognition.

3) (6p, **ONLY FINAL EXAM**) Consider a cascade of three LTI systems in Figure 5. It is known that

$$
\begin{aligned}
h_1[n] &= \delta[n-1] - \delta[n-2] \\
h_2[n] &= \delta[n+1] - 2\delta[n] + \delta[n-1]
\end{aligned}
$$

a) What is the impulse response $h[n]$ of the whole system? Show intermediate steps.

b) If the output from the system $h[n]$ is

$$y[n] = 2\delta[n+1] - 8\delta[n] + 11\delta[n-1] - 4\delta[n-2] - 4\delta[n-3] + 4\delta[n-4] - \delta[n-5]$$

what has been the input $x[n]$?

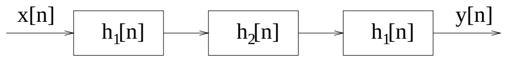

Figure 5: Problem 3: cascade system.

4) (6p, **ONLY FINAL EXAM**) Consider a digital LTI filter whose frequency response is

$$H(e^{j\omega}) = \frac{1 - 1.8e^{-j\omega} + 0.82e^{-2j\omega}}{1 + 0.9e^{-j\omega}}$$

a) Explain briefly whether the filter is FIR or IIR.

b) Sketch the pole-zero-plot of the filter.

c) Sketch the magnitude response of the filter. Is it lowpass / highpass / bandpass / bandstop / allpass?

d) Write down the difference equation of the filter.

e) Explain briefly whether the filter is causal or not.

f) Explain or draw one and only one other essential thing about the filter.

5) (6p, **ONLY FINAL EXAM**) Consider a LTI system in Figure 6.

a) Determine the transfer function $H(z)$ of the filter.

b) Draw the filter structure in "direct form I" and in "direct form II".

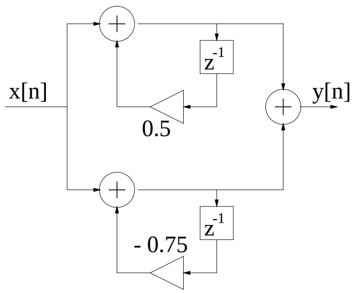

Figure 6: Problem 5, filter $H(z)$

6) (6p, **ONLY FINAL EXAM**) Consider an analog first order Butterworth highpass filter with cut-off at Ω_c:

$$H(s) = \frac{s}{s + \Omega_c}$$

Design a digital first order Butterworth highpass filter $H(z)$ by bilinear transform, when the cut-off is $f_c = 4000$ Hz and the sampling frequency is $f_T = 10000$ Hz. Apply "prewarping" for frequency distortion and express the transfer function in the format

$$H(z) = K \cdot \frac{1 + b_1 z^{-1} + \ldots + b_M z^{-M}}{1 + a_1 z^{-1} + \ldots + a_N z^{-N}}$$

7) (6p, **ONLY FINAL EXAM**) See the filter in Figure 7. The input values are represented with B bits. After multiplications the number of bits is $2B$. In order to get the number of bits in output to B, it is necessary to quantize values of $w[n]$ (block Q).

Quantization error can be compensated using so called error feedback (or error-shaping filter). In Figure 7 there is a second order filter with a second order error feedback system.

Write down first the difference equations for $e[n]$ and $w[n]$, and write down then in frequency domain the quantized output $Y(z)$ using input $X(z)$, filter modifing input $H_x(z)$, quantization noise $E(z)$, and filter modifing noise $H_e(z)$ in form of

$$Y(z) = H_x(z)X(z) + H_e(z)E(z)$$

and reply

a) how does the filter behave, if it is possible to use infinite wordlength, i.e. there is no quantization and $e[n] \equiv 0, \forall n$?

b) how does the spectrum of the total noise $E_{tot}(z) = H_e(z)E(z)$ look like if there is no compensation, i.e. $k = 0$, and if $e[n]$ is white noise so that $E(z) = 1$ for all frequencies?

c) with which simple value of k the effect of noise is suppressed in the passband?

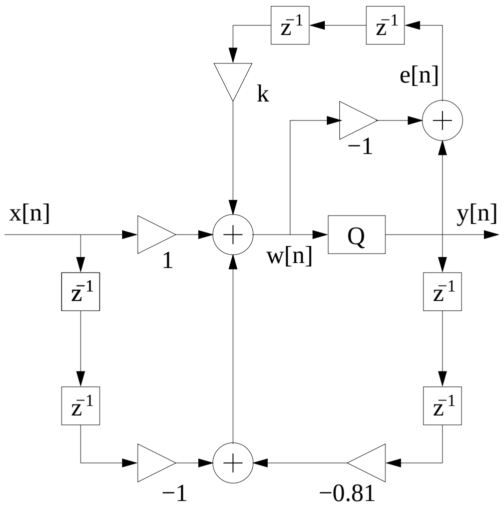

Figure 7: Problem 7. Second order system with second order error feedback.