T-61.3010 Digital Signal Processing and Filtering

2nd mid term exam / final exam, Mon 7-May-2007 at 8-11. Hall M (non-Finnish).

If you are doing **the 2nd mid term exam**, reply to problems **1-2.** You are allowed to do 2nd MTE **only once either 7.5. or 15.5.**

If you are doing **the final exam**, reply to problems **2-5.** You are allowed to do final exam **only once either 7.5. or 15.5.**

Write down on first paper, if you are doing **mid term exam or final exam!**.

You are not allowed to use any math table books of your own. A table of formulas is delivered as well as a form for multichoice problem (either 1 or 5). A (graphical) calculator is allowed.

Start a new task from a **new page**. Write all **intermediate steps**.

1) (**ONLY 2nd MTE**, 14 x 1p, max 12 p) Multichoice. There are 1-4 correct answers in the statements, but choose **one and only one.** Fill in your solutions in a specific **form**.

Correct answer +1 p, wrong answer −0.5 p, no answer 0 p. You do not need write why you chose your option. Reply to as many as you want. The maximum number of points is 12, and the minimum 0.

1.1 Which filter has linear phase response?
- **(A)** $H(z) = 1/[1 + 2z^{-1} - 3z^{-2} + 2z^{-3} + z^{-4}]$
- **(B)** $h[n] = (0.5)^n \mu[n]$
- **(C)** $h[n] = 0.42 + 0.5 \cdot \cos(\pi n/M) + 0.08 \cdot \cos(2\pi n/M)$, when $-M \leq n \leq M$, $M \in \mathbb{Z}_+$, and $h[n] = 0$, when $n < -M$ tai $n > M$.
- **(D)** $h[n] = 0.6\delta[n] + 0.4\delta[n-1]$

1.2 The LTI filter in Figure 1(a) with constants A and B
- **(A)** can be linear-phase if choosing A and B correctly from a set of positive integers
- **(B)** transfer function is $H(z) = 1/[1 - (A + B)z^{-1}]$
- **(C)** zeros of the filter are at $z_1 = -A$ and $z_2 = -B$
- **(D)** none of choices above is correct

1.3 The filter in Figure 1(b), where $H_1(z) = 1 + z^{-1}$ and $H_2(z) = 1 - z^{-1}$,
- **(A)** has group delay $\tau(\omega) = 1.5$, $\forall \omega$
- **(B)** has not linear phase
- **(C)** is lattice structure
- **(D)** is of type IIR

1.4 Consider a first order IIR filter

$$H(z) = \frac{1-\alpha}{2} \cdot \frac{1 - z^{-1}}{1 - \alpha z^{-1}}$$

where $-0.8 \leq \alpha \leq 0.8$. Choose α so that the maximum of the amplitude response $|H(e^{j\omega})|$ gets its greatest value. After that the filter is scaled by a constant K so that the amplitude response of the scaled filter is 1. Now
- **(A)** $K = 0.8$
- **(B)** $K = 1$
- **(C)** $K = 3$
- **(D)** none of choices above is correct

1.5 Bilinear transform:
- **(A)** right half of s-plane maps inside the unit circle
- **(B)** frequency axis $j\Omega$ in s-plane maps to y-axis in z-plane
- **(C)** unstable filter in s-plane is also unstable filter in z-plane
- **(D)** can be done using substitution $s = \frac{2}{T}\frac{1+z^{-1}}{1-z^{-1}}$, where T is sampling period

1.6 Analog filter $H(s) = \Omega/(s - \Omega)$, where prewarped cut-off frequency $\Omega = k \cdot 0.5$, is converted to digital $H(z)$ using bilinear transform. The digital filter is
- **(A)** $H(z) = (-1)/(1 - 2k^{-1}z^{-1})$
- **(B)** $H(z) = (1/3) \cdot (1 + z^{-1})/(1 - (1/3)z^{-1})$
- **(C)** $H(z) = (1 + z^{-1})/(1 - z^{-1})$
- **(D)** $H(z) = (1 + z^{-1})/(1 - 3z^{-1})$

1.7 In Matlab one wants to design an elliptic highpass filter, whose stopband ends at 4000 Hz and passband starts from 5000 Hz. The sampling frequency is 20000 Hz. For the command `ellipord` one has to normalize frequencies for Matlab. The correct command is:
- **(A)** `[N, Wn] = ellipord(2*5000*pi, 2*4000*pi, 1, 40, 20000);`
- **(B)** `[N, Wn] = ellipord(0.2, 0.25, 1, 40, 'high');`
- **(C)** `[N, Wn] = ellipord(0.5, 0.4, 1, 40);`
- **(D)** `[N, Wn] = ellipord(4000, 5000, 10000, 'HP');`

1.8 The ideal highpass filter $H_{HP}(z)$ has cut-off freuquency at $\omega_c = 3\pi/4$. Now
- **(A)** $h_{HP}[0] = 0$
- **(B)** $h_{HP}[0] = 0.25$
- **(C)** $h_{HP}[0] = 0.75$
- **(D)** $h_{HP}[0] = 1$

1.9 The impulse response $h_{HP}[n]$ of statement 1.8 is converted to a lowpass filter by multiplying (modulating) the sequence with $(-1)^n$, that is, $h[n] = h_{HP}[n] \cdot (-1)^n$. What is the cut-off frequency of the filter $h[n]$?
- **(A)** $\omega_c = \pi/4$
- **(B)** $\omega_c = \pi/2$
- **(C)** $\omega_c = 3\pi/4$
- **(D)** $\omega_c = \pi$

1.10 Discrete Fourier transform (DFT) can be computed efficiently by utiziling symmetry properties. In "bonus exercises" there was "radix-2 DIT FFT with modified butterfly computational module" algorithm, which is one of many FFT algorithms.

In Figure 3 there is the graph of computation, when the length of the sequence to be transformed is $N = 128$. The butterfly equations are:

$$\begin{aligned} \Psi_{r+1}[\alpha] &= \Psi_r[\alpha] + W_N^l \Psi_r[\beta] \\ \Psi_{r+1}[\beta] &= \Psi_r[\alpha] - W_N^l \Psi_r[\beta] \end{aligned}$$

where $W_N = e^{-j2\pi/N}$.

The sequence to be transformed is $x[n] = \{\underline{0}, 1, 2, 3, \ldots, 127\} = n$. What can be said about the term $\Psi_3[87]$?
- **(A)** $\Psi_3[87] = -1 - j$
- **(B)** $\Psi_3[87] = -64 + 64j$
- **(C)** $\Psi_3[87] = -64 - 64j$
- **(D)** $\Psi_3[87] = 85 - 117j$

1.11 Examine the same structure and sequence as in statement 1.10. Which of the first level values $\Psi_1[0], \ldots, \Psi_1[127]$ affect values of the "output layer" term $\Psi_8[87]$?
- **(A)** only $\Psi_1[87]$
- **(B)** only $\Psi_1[23]$ and $\Psi_1[87]$
- **(C)** all $\Psi_1[n]$, where $n = 0, \ldots, 127$
- **(D)** none of choices above is correct

1.12 Examine the same structure and sequence as in statement 1.10. Compute the sum $S = \Psi_6[0] + \Psi_6[32] + \Psi_6[64] + \Psi_6[96]$.
- **(A)** $S = 6$
- **(B)** $S = 192$
- **(C)** $S = 8128$
- **(D)** none of choices above is correct

1.13 Examine the spectrum $|X(e^{j\omega})|$ of a real sequence $x[n]$ in Figure 2(a). The sequence is fed into multirate system $x[n] \rightarrow \boxed{\downarrow 4} \rightarrow y[n]$.

What can be said about the spectrum of the output (assumption: scales in y-axis correct)?
- **(A)** none of the choices below is true
- **(B)** spectrum of the output $y[n]$ is in Figure 2(b)
- **(C)** spectrum of the output $y[n]$ is in Figure 2(c)
- **(D)** spectrum of the output $y[n]$ is in Figure 2(d)

1.14 The sampling frequency of a digital sequence is to be increased to (7/5) of the original. When having proper "anti-alias" and "anti-imaging" filters $H_i(z)$, what is the correct way to do this?
- **(A)** $x[n] \rightarrow \boxed{\uparrow 7} \rightarrow \boxed{H(z)} \rightarrow \boxed{\downarrow 5} \rightarrow y[n]$
- **(B)** $x[n] \rightarrow \boxed{\downarrow 5} \rightarrow \boxed{H(z)} \rightarrow \boxed{\uparrow 7} \rightarrow y[n]$
- **(C)** $x[n] \rightarrow \boxed{H_D(z)} \rightarrow \boxed{\downarrow 5} \rightarrow \boxed{\uparrow 7} \rightarrow y[n]$
- **(D)** $x[n] \rightarrow \boxed{\uparrow 7} \rightarrow \boxed{\downarrow 5} \rightarrow \boxed{H_I(z)} \rightarrow y[n]$

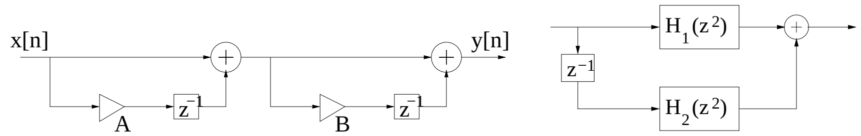

Figure 1: (a) and (b): Figures for multichoice statements 1.2 and 1.3.

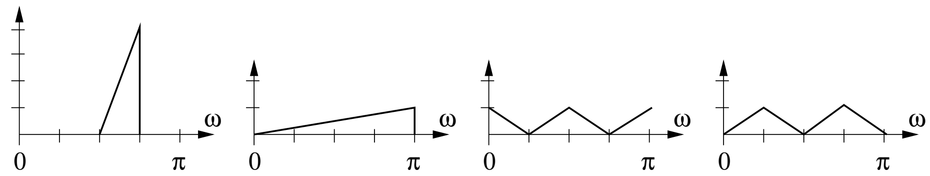

Figure 2: (a): $|X(e^{j\omega})|$, (b) **(B)** , (c) **(C)** ja (d) **(D)** : Figures for multichoice statement 1.13.

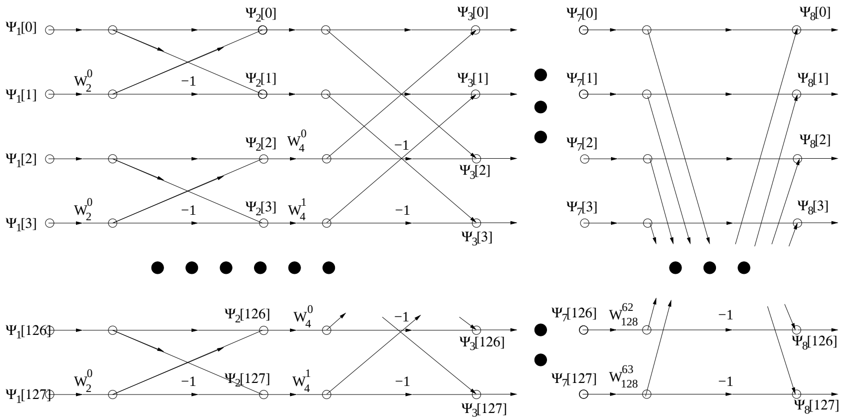

Figure 3: Figures for multichoice statements 1.10 – 1.12 and 5.12 – 5.13.

2) (**MTE and FINAL EXAM**, 6p) Essay: Finite wordlength and its effects.

Instructions: Write down clearly with big enough font. Divide your text into paragraphs. If you draw figures, remember to explain them.

3) (**ONLY FINAL EXAM**, 6p) Speech signal with sampling frequency $f_T = 20$ kHz is analyzed and found out to have useful signal at frequencies $0 \ldots 7000$ Hz and the noise at $8000 \ldots f_T/2$ Hz. Let us design a FIR-type lowpass filter with window method. The required -6 dB cut-off frequency, which corresponds the cut-off of the ideal filter, is defined to be $f_c = 7500$ Hz and the length of the transition band is $\Delta f = 1000$ Hz. Apply Blackman window function whose parameters are given in Table 1.

a) Increasing the order of the filter narrows the transition band. Estimate a correct order N using Table 1.

b) Write down the expression for the ideal lowpass filter $h_{\text{ideal}}[n]$. Compute and write down with three significant digits (e.g. -140, 2.31, 0.00621) the values $h_{\text{ideal}}[-7]$, $h_{\text{ideal}}[0]$, $h_{\text{ideal}}[7]$, and $h_{\text{ideal}}[2007]$.

c) Using Blackman function implement a FIR filter $h_{\text{FIR}}[n]$ with the order computed in (a). Compute and write down with three significant digits the values $h_{\text{FIR}}[-7]$, $h_{\text{FIR}}[0]$, $h_{\text{FIR}}[7]$, and $h_{\text{FIR}}[2007]$.

Window	$w[n]$, $-M \le n \le M$	Mininum stopband attenuation	Length of transition band $\Delta\omega$
Blackman	$0.42 + 0.5\cos(\frac{2\pi n}{2M}) + 0.08\cos(\frac{4\pi n}{2M})$	75.3 dB	$5.56\pi/\mathrm{M}$

Table 1: Problem 3: Properties of Blackman window function.

4) (**ONLY FINAL EXAM**, 6p) Examine the cascade system of three LTI systems in Figure 4. It is known that $h_1[n] = \delta[n-1] - \delta[n-2]$ and the whole cascade system $h[n] = 2\delta[n] - 5\delta[n-1] + 5\delta[n-2] - 3\delta[n-3] + \delta[n-4]$.

a) What is the impulse response of $h_2[n]$? Remember to give the intermediate steps.

b) Draw the pole-zero-plots of $h_1[n]$ and $h_2[n]$ and sketch also the corresponding amplitude responses $|H_i(e^{j\omega})|$.

c) What can be said about stability and causality of systems $h_1[n]$, $h_2[n]$ and $h[n]$ based on the definitions of those. Show.

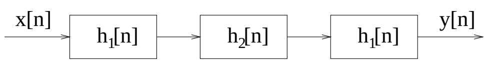

Figure 4: Problem 4 cascade system

5) (**ONLY FINAL EXAM**, 14 x 1p, max 12 p) Multichoice. There are 1-4 correct answers in the statements, but choose **one and only one.** Fill in your solutions in a specific **form**.

Correct answer +1 p, wrong answer -0.5 p, no answer 0 p. You do not need write why you chose your option. Reply to as many as you want. The maximum number of points is 12, and the minimum 0.

5.1 Examine discrete-time system $y[n] = 4x[3n+2] + 1$:
(A) system is linear
(B) system is time-invariant
(C) system is causal
(D) none of choices above is correct

5.2 The fundamental period N_0 of sequence $x[n] = \cos(\pi n/3) + 2\sin(0.25\pi n) - \sin(2\pi n/16)$
(A) $N_0 = 2$
(B) $N_0 = 6$
(C) $N_0 = 48$
(D) $N_0 = 768$

5.3 The sequence $x[n] = \cos(0.25\pi n^2)$
(A) is not periodic
(B) is periodic but the length of fundamental period is ∞
(C) fundamental period $N_0 = 8$
(D) none of choices above is correct

5.4 What is the value $h[3]$ of the inverse transform $h[n]$ of the causal and stable transfer function $H(z) = \frac{1-z^{-2}}{1+0.5z^{-1}}$ with three significant digits?
(A) $h[3] = -0.952$
(B) $h[3] = -0.375$
(C) $h[3] = 0.375$
(D) $h[3] = 0.952$

5.5 See the spectrum $|X(j\Omega)|$ of continous-time signal in top row of Figure 5. The sampling frequency is $f_T = 10$ kHz. The spectrum of the sequence $x[n]$ is in bottom row of Figure 5
(A) (a)
(B) (b)
(C) (c)
(D) (d)

5.6 In order to avoid aliasing in the sampling process, the sampling period T_s has to be
(A) at least ten times larger than the highest frequency component in the signal
(B) at least two times as long as the fundamental period T_0 of the highest frequency component
(C) at most half of the fundamental period T_0 of the highest frequency component
(D) none of choices above is correct

5.7 The pole-zero diagram of a LTI system $H(z)$ in Figure 6(a) corresponds the magnitude response
(A) in Figure 7(a)
(B) in Figure 7(b)
(C) in Figure 7(c)
(D) in Figure 7(d)

5.8 Magnitude response of a LTI filter $H(z)$ in Figure 6(b) corresponds the pole-zero diagram
(A) in Figure 8(a)
(B) in Figure 8(b)
(C) in Figure 8(c)
(D) in Figure 8(d)

5.9 The frequency response of a filter is $H(e^{j\omega}) = 1 + 0.1e^{-8j\omega}$
(A) Impulse response is 8 samples long
(B) Zeros of the filter are on the same circle with equal intervals
(C) Phase response is linear
(C) Filter is lowpass filter even if the curve of magnitude response does not decrease monotonically
(D) Order of the filter is 2

5.10 Examine transfer function $H(z) = 1/(1 - 5z^{-1} + 6z^{-2})$. Choosing ROC, "region of convergence"
(A) $|z| < 2$ the filter $h[n]$ is stable
(B) $|z| < 3$ the filter $h[n]$ is stable
(C) $|z| > 3$ the filter $h[n]$ is stabiili and also causal
(D) way or another – it is impossible to get both causal and stable filter $h[n]$ at the same time

5.11 Consider a transfer function $H(z) = 1/(1 - 5z^{-1} + 6z^{-2})$. Choosing a region of convergence
(A) $|z| < 2$ there will be a stable filter $h[n]$
(B) $|z| < 3$ there will be a stable filter $h[n]$
(C) $|z| > 3$ there will be a stable $h[n]$, which is also causal
(D) way or another, it is impossible to get both causal and stable filter at the same time

5.12 Discrete Fourier transform (DFT) can be computed efficiently by utiziling symmetry properties.
In Figure 3 there is graph of computation of "radix-2 DIT FFT with modified butterfly computational module", which is one of many FFT algorithms. The length of the sequence to be transformed is $N = 128$. So called butterfly equations are:

$$\begin{aligned} \Psi_{r+1}[\alpha] &= \Psi_r[\alpha] + W_N^l \Psi_r[\beta] \\ \Psi_{r+1}[\beta] &= \Psi_r[\alpha] - W_N^l \Psi_r[\beta] \end{aligned}$$

where $W_N = e^{-j2\pi/N}$.
The sequence to be transformed is $x[n] = \{\underline{0}, 1, 2, 3, \ldots, 127\} = n$. What can be said about the term $\Psi_3[73]$?
(A) $\Psi_3[73] = -1 + j$
(B) $\Psi_3[73] = -64 + 64j$
(C) $\Psi_3[73] = -64 - 64j$
(D) $\Psi_3[73] = -74 + 105j$

5.13 Examine the same structure and sequence as in statement 5.12. Compute the sum
$S = \Psi_6[0] + \Psi_6[32] + \Psi_6[64] + \Psi_6[96]$.
(A) $S = 6$
(B) $S = 192$
(C) $S = 8128$
(D) none of choices above is correct

5.14 The sampling frequency of a digital sequence is to be increased to (7/5) of the original. When having proper "anti-alias" and "anti-imaging" filters $H_i(z)$, what is the correct way to do this?

(A) $x[n] \rightarrow \boxed{\uparrow 7} \rightarrow \boxed{H(z)} \rightarrow \boxed{\downarrow 5} \rightarrow y[n]$

(B) $x[n] \rightarrow \boxed{\downarrow 5} \rightarrow \boxed{H(z)} \rightarrow \boxed{\uparrow 7} \rightarrow y[n]$

(C) $x[n] \rightarrow \boxed{H_D(z)} \rightarrow \boxed{\downarrow 5} \rightarrow \boxed{\uparrow 7} \rightarrow y[n]$

(D) $x[n] \rightarrow \boxed{\uparrow 7} \rightarrow \boxed{\downarrow 5} \rightarrow \boxed{H_I(z)} \rightarrow y[n]$

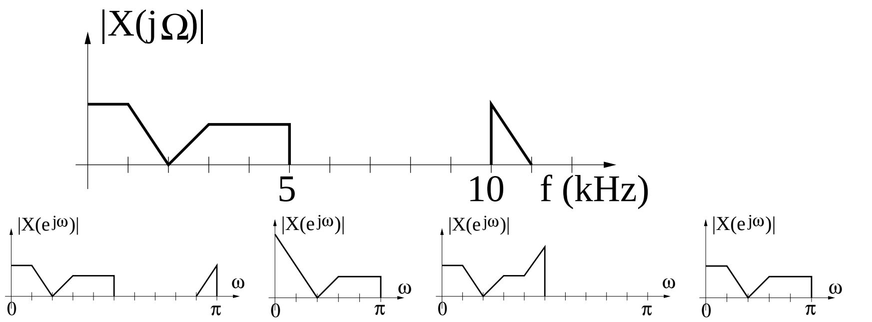

Figure 5: Figures for multichoice statement 5.5. Top row: continuous-time $X(j\Omega)$, bottom row: choices **(A)** , **(B)** , **(C)** , **(D)**

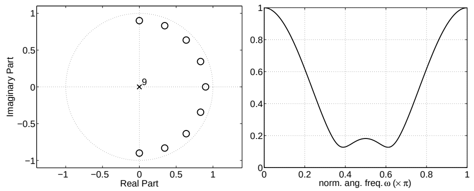

Figure 6: (a) and (b): Figures for multichoice statements 5.7 and 5.8.

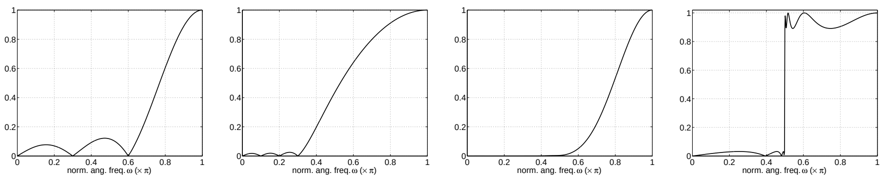

Figure 7: (a), (b), (c) and (d): Figures for multichoice statement 5.7.

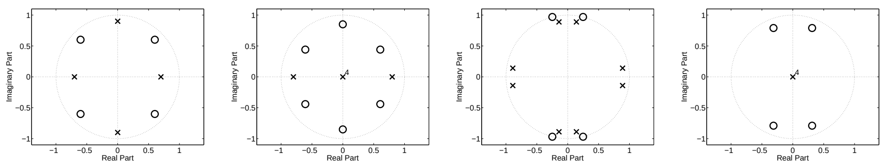

Figure 8: (a), (b), (c) and (d): Figures for multichoice statement 5.8.