T-61.3010 Digital Signal Processing and Filtering

2nd mid term exam / final exam, Tue 9th May 2006 at 8-11. Hall M (non-Finnish).

If you are doing **the 2nd mid term exam**, reply to problems **1-3**. You are allowed to do 2nd MTE **only once either 9.5. or 16.5.**

If you are doing **the final exam**, reply to problems **2-5**. You are allowed to do final exam **only once either 9.5. or 16.5.**

Write down on first paper, if you are doing **mid term exam or final exam!**.

You are not allowed to use any math table books of your own. A table of formulas is delivered as well as a form for multichoice problem (either 1 or 5). A (graphical) calculator is allowed.

Start a new task from a **new page**. Write all **intermediate steps**.

1) (**ONLY 2nd MTE**, 14 x 1p, max 12 p) Multichoice. There are 1-4 correct answers in the statements, but choose **one and only one.** Fill in your solutions in a specific **form**.

Correct answer +1 p, wrong answer −0.5 p, no answer 0 p. You do not need write why you chose your option. Reply to as many as you want. The maximum number of points is 12, and the minimum 0.

1.1 The filter in Figure 1(a):

   (**A**) is of type Direct Form I

   (**B**) is of FIR type

   (**C**) the transfer function is $H(z) = \frac{1+0.5z^{-1}+0.2z^{-2}}{1-0.1z^{-1}-2z^{-2}}$

   (**D**) the number of delay elements is the same as the order of the filter

1.2 The filter in Figure 1(b)

   (**A**) is an allpass filter with a certain constant $K > 0$

   (**B**) is of type Direct Form I

   (**C**) is of type lattice

   (**D**) is unstable

1.3 The filter in Figure 2(a), where $H_1(z) = 1 + 0.5z^{-1}$ and $H_2(z) = 0.5 + z^{-1}$,

   (**A**) has group delay one

   (**B**) has linear phase

   (**C**) is of type lattice

   (**D**) is of type IIR

1.4 Consider a block diagram in Figure 2(b), where A and B are positive constants

   (**A**) A sequence $y[n]$ is $y[n] = B \cdot (Ax[n] + w[n])$

   (**B**) It is not possible to build the filter physically

   (**C**) The filter can be represented as a Direct Form I with feedback

   (**D**) The filter is of type FIR

1.5 Consider a block diagram in Figure 2(c), where three subsystems $S(z)$, $R(z)$ and $T(z)$ are LTI. The transfer function of the filter is

   (**A**) $H(z) = \frac{S(z)+R(z)T(z)}{1-T(z)}$

   (**B**) $H(z) = \frac{S(z)R(z)}{1-S(z)T(z)}$

   (**C**) $H(z) = \frac{R(z)S(z)}{1+S(z)T(z)}$

   (**D**) $H(z) = \frac{R(z)S(z)}{T(z)}$

1.6 In Matlab the command `ellipord` is used to compute the order and the cut-off of the elliptic filter. What to type in Matlab in order to get more information?

   (**A**) `help ellipord`

   (**B**) `man ellipord`

   (**C**) `info ellipord`

   (**D**) `doc ellipord`

1.7 In Matlab one wants to design an elliptic high-pass filter, whose stopband ends at 2000 Hz and passband starts from 3000 Hz. The sampling frequency is 10000 Hz. For the command `ellipord` on has to normalize frequencies for Matlab. The correct command is:

   (**A**) `[N, Wn] = ellipord(0.2, 0.3, 1, 40, 'high');`

   (**B**) `[N, Wn] = ellipord(2000, 3000, 10000, 'HP');`

   (**C**) `[N, Wn] = ellipord(0.6, 0.4, 1, 40);`

   (**D**) `[N, Wn] = ellipord(2*3000*pi, 2*2000*pi, 1, 40, 10000);`

1.8 The analog filter $H(s) = 1/(s+0.5)$ is converted to the digital one $H(z)$ using bilinear transform. The digital filter is

   (**A**) $H(z) = 1/(z^{-1} + 0.5)$

   (**B**) $H(z) = 2/(1 + 2z^{-1})$

   (**C**) $H(z) = (2/3) \cdot (1 + z^{-1})/(1 - (1/3)z^{-1})$

   (**D**) $H(z) = (1 + z^{-1}) \cdot (1 - z^{-1})/(1 + 0.5z^{-1})$

1.9 Fast Fourier Transform (FFT)

   (**A**) computes discrete Fourier transform (DFT) always more accurate if there are more computing power available

   (**B**) computes discrete Fourier transform (DFT) with less operations than with definition of DFT

   (**C**) sequence can be extended with zeros (zero padding) to length of 2^N so that FFT can be used

   (**D**) requires some N^2 complex operations with large N

1.10 Effects of finite wordlength can be caused by

   (**A**) quantizing filter coefficients

   (**B**) rounding multiplication results

   (**C**) truncating multiplication results

   (**D**) quantizing analog input

1.11 Quantization error feedback filter ("error spectrum shaping"):

   (**A**) It would be not needed at all if there were infinite accuracy in computation

   (**B**) Quantization noise can be suppressed to a band of no interest

   (**C**) It has to be used always in order to have any LTI filter work

   (**D**) Quantization error $e[n]$ is modified and fed back into system

1.12 Periodic sequence $(N_0 = 6)$ $x[n] = \{\ldots, 3, 8, 4, 2, 9, -1, \ldots\}$ is fed into digital system $x[n] \rightarrow \boxed{\downarrow 2} \rightarrow \boxed{\uparrow 2} \rightarrow y[n]$. What comes out?

   (**A**) the same sequence $y[n] = x[n]$

   (**B**) sequence $y[n]$ but with half length

   (**C**) $y[n] = \{\ldots, 3, 0, 4, 0, 9, 0, \ldots\}$

   (**D**) $y[n] = 2x[n]$

1.13 Periodic sequence $(N_0 = 4)$ $x[n] = \{\ldots, 7, 3, -2, 8, \ldots\}$ is fed into digital system $x[n] \rightarrow \boxed{\uparrow 3} \rightarrow \boxed{\downarrow 3} \rightarrow y[n]$. What comes out?

   (**A**) the same sequence $y[n] = x[n]$

   (**B**) $y[n] = \{\ldots, 7, 0, 0, 8, 7, 0, 0, 8, \ldots\}$, where period $N_0 = 4$

   (**C**) $y[n] = \{\ldots, 7, 0, 0, 8, 0, 0, -2, 0, 0, 3, 0, 0, \ldots\}$, where period $N_0 = 12$

   (**D**) $y[n] = 3x[n]$

1.14 The sampling frequency of a digital sequence is to be decreased to $(2/5)$ of the original. When having proper "anti-alias" and "anti-imaging" filters $H_i(z)$, what is the correct way to do this?

   (**A**) $x[n] \rightarrow \boxed{\uparrow 5} \rightarrow \boxed{H(z)} \rightarrow \boxed{\downarrow 2} \rightarrow y[n]$

   (**B**) $x[n] \rightarrow \boxed{\uparrow 2} \rightarrow \boxed{H(z)} \rightarrow \boxed{\downarrow 5} \rightarrow y[n]$

   (**C**) $x[n] \rightarrow \boxed{H_I(z)} \rightarrow \boxed{\uparrow 2} \rightarrow \boxed{\downarrow 5} \rightarrow y[n]$

   (**D**) $x[n] \rightarrow \boxed{H_D(z)} \rightarrow \boxed{\downarrow 5} \rightarrow \boxed{\uparrow 2} \rightarrow y[n]$
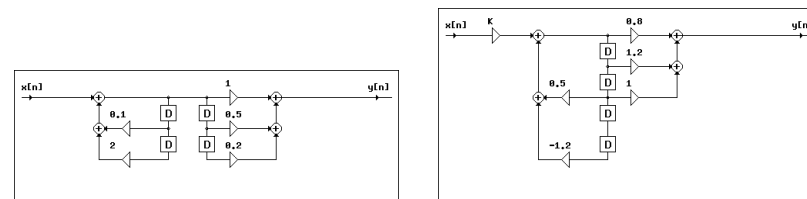

Figure 1: (a) and (b): Figures for multichoice statements 1.1 and 1.2.

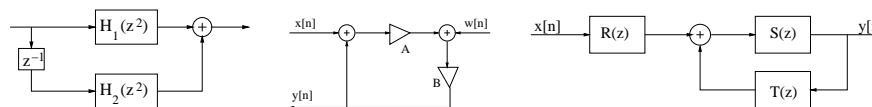
Figure 2: (a), (b) and (c): Figures for multichoice statements 1.3, 1.4 and 1.5.

2) (**MTE and FINAL EXAM**, 6p) Consider a flow diagram in Figure 3, which generates two sequences $y_1[n]$ and $y_2[n]$ with given constant ω_0. Which functions (sequences) are generated, when the input is $x[n] = A\delta[n]$. Write down clear steps.
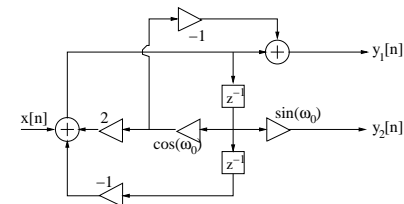
Hint: See z-transform tables!

Figure 3: Flow diagram of Problem 2.

3) (**MTE and FINAL EXAM**, 6p) Design a digital FIR filter with window method ("FIR filter design method based on windowed Fourier series"). It has to be **lowpass** which cut-off frequency $f_c = 1000$ Hz and the sampling frequency $f_T = 8000$ Hz. Compute fourth order **causal** filter and use Hamming window

$$w_{Hamming}[n] = 0.54 + 0.46\cos(\frac{2\pi n}{2M}) \qquad -M \le n \le +M$$

Write down clear steps and compute exact values for causal impulse response $h[0], \ldots, h[4]$, when the maximum of the filter is scaled to one.

4) (**ONLY FINAL EXAM**, 6p) Difference equation of LTI filter is of form

$$y[n] = x[n] - x[n-1] - 2a\, y[n-1] - 2a^2\, y[n-2]$$

where coefficient a is a real constant, whose value is defined later.

  a) Draw the block (flow) diagram of the filter.

  b) Determine the transfer function of the filter (you do not have to care about regions of convergence (ROC)).

  c) Examine the frequency properties of the causal filter as a function of a. Use analysis of pole-zero diagram, when $a \ge 0$.

5) (**ONLY FINAL EXAM**, 14 x 1p, max 12 p) Multichoice. There are 1-4 correct answers in the statements, but choose **one and only one.** Fill in your solutions in a specific **form**.

Correct answer +1 p, wrong answer −0.5 p, no answer 0 p. You do not need write why you chose your option. Reply to as many as you want. The maximum number of points is 12, and the minimum 0.

5.1 Two-point moving average filter

  (**A**) is of IIR type

  (**B**) has transfer function $H(z) = 0.5(\delta[n]-\delta[n-1])$

  (**C**) attenuates quick changes in the signal

  (**D**) difference equation is $y[n] = 0.5(x[n]+x[n-1])$

5.2 The fundamental period N_0 of sequence $x[n] = \cos(\pi n) + 2\sin(0.25\pi n)$

  (**A**) $N_0 = 2$

  (**B**) $N_0 = 4$

  (**C**) $N_0 = 8$

  (**D**) $N_0 = 16$

5.3 The sequence $x[n] = \cos(0.5\pi n^2)$

  (**A**) is not periodic

  (**B**) is periodic but the length of fundamental period is ∞

  (**C**) fundamental period $N_0 = 2$

  (**D**) fundamental period $N_0 = 4$

5.4 Discrete-time filter, whose input-output relation is

$$y[n] = x[n] + (x[n+1] \cdot x[n-1])$$

  (**A**) is causal

  (**B**) is time-invariant

  (**C**) is linear

  (**D**) is BIBO-stable

5.5 Convolution of sequences $x[n] = \delta[n-1]+\delta[n+1]$ and $h[n] = \delta[n-2] + \delta[n+2]$ gives

  (**A**) $y[n] = \delta[n-3]+2\delta[n-1]+2\delta[n+1]+\delta[n+3]$

  (**B**) $y[n] = \delta[n-1] + \delta[n-3]$

  (**C**) $y[n] = \delta[n-3] + \delta[n+3]$

  (**D**) $y[n] = \delta[n-3]+\delta[n-1]+\delta[n+1]+\delta[n+3]$

5.6 Deconvolution of sequences $y[n] = \{\underline{-4},-2,4,-1\}$ and $h[n] = \{\underline{2},-1\}$ gives sequences $x[n]$

  (**A**) $x[n] = \{\underline{2},1\}$

  (**B**) $x[n] = \{\underline{2},-1,4,1\}$

  (**C**) $x[n] = \{\underline{-2},-2,1\}$

  (**D**) $x[n] = \{\underline{2},-2,-1\}$

5.7 What is the value $h[n]$ at $n = 3$, where $h[n]$ is inverse transform of transfer function $H(z) = \frac{1+z^{-3}}{1+0.2z^{-1}}$?

  (**A**) $h[3] = 0.032$

  (**B**) $h[3] = 0.9722$

  (**C**) $h[3] = 0.992$

  (**D**) $h[3] = 1.008$

5.8 The spectrum $|X(j\Omega)|$ of a bandlimited analog real signal is given in Figure 4(a). It is sampled with a lowest possible sampling frequency not causing aliasing. Which of Figures 4(b) ... 4(e) does fit as a spectrum $|X(e^{j\omega})|$ of sampled sequence?

  (**A**) Figure 4(b), in range $0 \ldots \pi$, middle row left

  (**B**) Figure 4(c), in range $-\pi \ldots \pi$, middle row right

  (**C**) Figure 4(d), in range $0 \ldots \pi$, bottom row left

  (**D**) Figure 4(e), in range $0 \ldots \pi$, bottom row right

5.9 Length of sequence $x[n]$ is 1000 after sampling with 20 kHz. If the sampling frequency had been 10 kHz, what would be the length of sequence?

  (**A**) 500

  (**B**) 1000

  (**C**) 2000

  (**D**) 10000

5.10 Consider a minimum-phase filter $H(z)$, for which the inverse filter is $1/H(z)$. What can be said about the inverse filter:

  (**A**) FIR

  (**B**) stable

  (**C**) zeros with equal steps on unit circle

  (**D**) always causal

5.11 Consider a transfer function $H(z) = 1/(1 - 5z^{-1} + 6z^{-2})$. Choosing a region of convergence

  (**A**) $|z| < 2$ there will be a stable filter $h[n]$

  (**B**) $|z| < 3$ there will be a stable filter $h[n]$

  (**C**) $|z| > 3$ there will be a stable $h[n]$, which is also causal

  (**D**) way or another, it is impossible to get both causal and stable filter at the same time

5.12 A filter $H_1(z) = \frac{1+0.4z^{-1}}{1-0.8z^{-1}}$ is known. What is $H_2(z) = H_1(z^2)$?

  (**A**) $H_2(z) = \frac{z^2 \cdot (1+0.4z^{-1})}{1-0.8z^{-1}}$

  (**B**) $H_2(z) = \frac{1+0.4z^2}{1-0.8z^2}$

  (**C**) $H_2(z) = \frac{1+0.4z^{-2}}{1-0.8z^{-2}}$

  (**D**) $H_2(z) = 0.4z^{-2} + 0.5 - 1.25z^2$

5.13 The transfer function of a (monotonic) highpass filter is $H(z) = K \cdot (1 - z^{-1})$. If the maximum of the filter is scaled to one, the coefficient K must be

  (**A**) $K = 1/\infty$

  (**B**) $K = 0.5$

  (**C**) $K = 1$

  (**D**) $K = 2$

5.14 The sampling frequency of a digital sequence is to be decreased to $(2/5)$ of the original. When having proper "anti-alias" and "anti-imaging" filters $H_i(z)$, what is the correct way to do this?

  (**A**) $x[n] \to \boxed{\uparrow 5} \to \boxed{H(z)} \to \boxed{\downarrow 2} \to y[n]$

  (**B**) $x[n] \to \boxed{\uparrow 2} \to \boxed{H(z)} \to \boxed{\downarrow 5} \to y[n]$

  (**C**) $x[n] \to \boxed{H_I(z)} \to \boxed{\uparrow 2} \to \boxed{\downarrow 5} \to y[n]$

  (**D**) $x[n] \to \boxed{H_D(z)} \to \boxed{\downarrow 5} \to \boxed{\uparrow 2} \to y[n]$
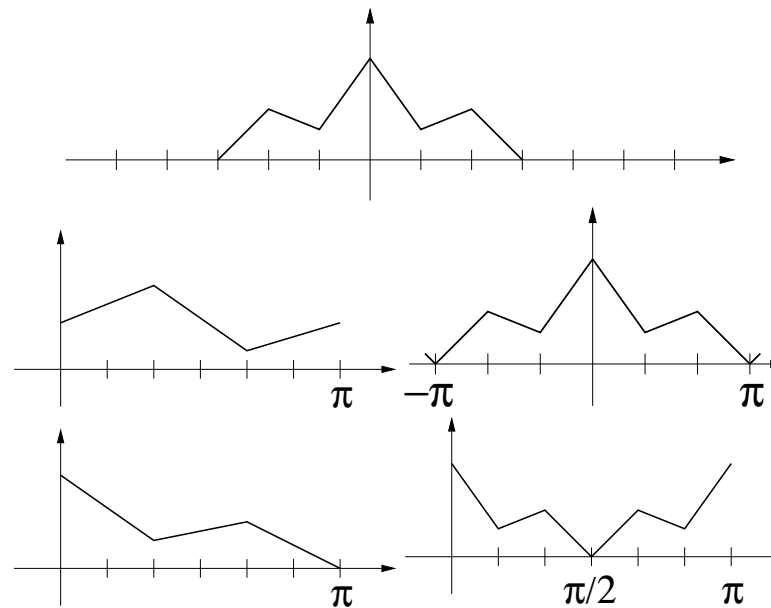

Figure 4: (a) top middle, (b), (c), (d), (e): Figures for multichoice statement 5.8.