T-61.3010 Digital Signal Processing and Filtering

1st mid term exam, Tue 7th March 2006 at 12-15. Hall M (N-Ö, non-Finnish).
You are allowed to do 1st MTE only once either 7.3. or 11.3.

You are not allowed to use any calculators or math tables. A table of formulas is delivered as well as a form for Problem 1.

All papers have to be returned. **In Problem 1 you must return a specific paper.** Problem statement and a paper of formulas you can keep.

Start a new task from a **new page**. Write all **intermediate steps**.

1) (14 x 1p, max 12 p) Multichoice. There are 1-4 correct answers in the statements, but choose **one and only one.** Fill in a specific **form**. The inspector has an eraser if needed.

   Correct answer +1 p, wrong answer −0.5 p, no answer 0 p. You do not need write why you chose your option. Reply to as many as you want. The maximum number of points is 12, and the minimum 0.

   1.1 It is possible to use spectrogram as a tool when analyzing speech signal.

   (A) In the spectrogram time runs in x-axis and frequency in y-axis

   (B) Spectrogram visualizes spectra of narrow time windows (short-time Fourier-transform)

   (C) In Matlab you can get the spectrogram figure with command `specgram(x, [], fs)`, where `x` is the sequence and `fs` sampling frequency.

   (D) With color (or gray scale) it is possible to represent the strength of each frequency component at any time.

   1.2 Two point moving averaging filter:

   (A) is IIR filter

   (B) transfer function $H(z) = 0.5(\delta[n] + \delta[n-1])$

   (C) suppresses quick changes in the signal

   (D) pole-zero diagram is in Figure 1(a)

   1.3 The impulse response of the filter is $h[n] = (-1)^{n+2}\mu[n+2]$:

   (A) filter is causal

   (B) filter is stable

   (C) filter is IIR

   (D) filter has 2 poles outside the unit circle

   1.4 Consider a sequence $x[n] = x_1[n]+x_2[n]+x_3[n]$, where fundamental periods of subsequences are $N_1 = 8$, $N_2 = 10$ and $N_3 = 20$. What can be said about period of sequence $x[n]$?

   (A) There is no fundamental period N_0

   (B) Fundamental period is $N_0 = 4$

   (C) Fundamental normalized angular frequency is $\omega_0 = \pi/200$

   (D) Sequence is periodic with period $N = 8 \cdot 10 \cdot 20 = 1600$

   1.5 The impulse response $h[n]$ of filter in Figure 2(a) convolved with input sequence $x[n] = 0.5^n\mu[n]$

   (A) produces a finite length output sequence

   (B) cannot be computed because filter is unstable

   (C) gets non-zero values from $n = 0$

   (D) produces output value $y[0] = 1$ at $n = 0$

1.6 Imaginary exponential function $e^{j\omega}$:
- **(A)** draws a unit circle, when $\omega = [0 \ldots \pi]$
- **(B)** can be written as $e^{j\omega} = \cos(\omega) + \sin(\omega)$
- **(C)** the real part is cosine function
- **(D)** absolute value of that is the filter phase response

1.7 Consider a filter $H(z) = 1/(1 - 0.5z^{-8})$.
- **(A)** The amplitude response is in Figure 1(b)
- **(B)** The pole-zero diagram is in Figure 1(c)
- **(C)** Filter is second-order IIR
- **(D)** Impulse repsonse $h[n]$ is nine units long

1.8 Transfer function $H(z) = [1 - 0.3z^{-1} + z^{-2}]/[1 - z^{-2}]$.
- **(A)** Filter is of fourth order
- **(B)** Flow/Block diagram is in Figure 2(b)
- **(C)** Impulse response is $h[n] = \delta[n] - 0.3\delta[n-1] + \delta[n-2]$
- **(D)** Magnitude response is in Figure 1(d)

1.9 Pole-zero plot corresponding amplitude response in Figure 1(e)
- **(A)** is in Figure 1(f)
- **(B)** is in Figure 1(g)
- **(C)** contains a zero at $z = 1$
- **(D)** contains a zero at $\omega = 0$

1.10 Consider $H(z)$ with four poles at $p_1 = 0.8e^{j0.25\pi}$, $p_2 = 1.25e^{j0.25\pi}$, $p_3 = 0.8e^{-j0.25\pi}$, $p_4 = 1.25e^{-j0.25\pi}$. Choosing the region of convergence (ROC) $0.8 < |z| < 1.25$ it is possible to say that
- **(A)** filter is stable
- **(B)** maximum amplification of filter is 2
- **(C)** phase response is linear
- **(D)** numerator polynomial of transfer function is of fourth order

1.11 Consider a filter in Figure 2(c)
- **(A)** filter is lowpass
- **(B)** difference equation is $y[n] = x[n] + 1.3x[n-1] + 0.4x[n-2]$
- **(C)** filter order is 2
- **(D)** transfer function is $H(z) = [1 + 0.8z^{-1}]/[1 + 0.5z^{-1}]$

1.12 Filter $H(e^{j\omega}) = e^{j\omega} - e^{j5\omega}$
- **(A)** is causal
- **(B)** poles are outside the unit circle
- **(C)** group delay is 3
- **(D)** has linear phase

1.13 Consider a LTI system with impulse response $h[n] = (-1)^{n-2}\mu[n+2]$ and input $x[n] = 2\delta[n+3] - 3\delta[n+2] + \delta[n+1]$. Output $y[n] = h[n] \circledast x[n]$ is computed.
- **(A)** $y[2006] = -6$
- **(B)** $y[2006] = 0$
- **(C)** $y[2006] = 6$
- **(D)** $y[2006] = -2\delta[n+2009] - 3\delta[n+2008] + \delta[n+2007]$

1.14 In the series (cascade) connection of two LTI systems S_1 and S_2

(A) the pole-zero plot of the whole system S is received by computing poles and zeros from both subsystems and drawing them into the same pole-zero-plot.

(B) impulse response of the whole system S is derived by summing impulse responses of subsystems

(C) impulse response of the whole system S is derived by multiplication of impulse responses of subsystems

(D) impulse response of the whole system S is derived by convolving impulse responses of subsystems

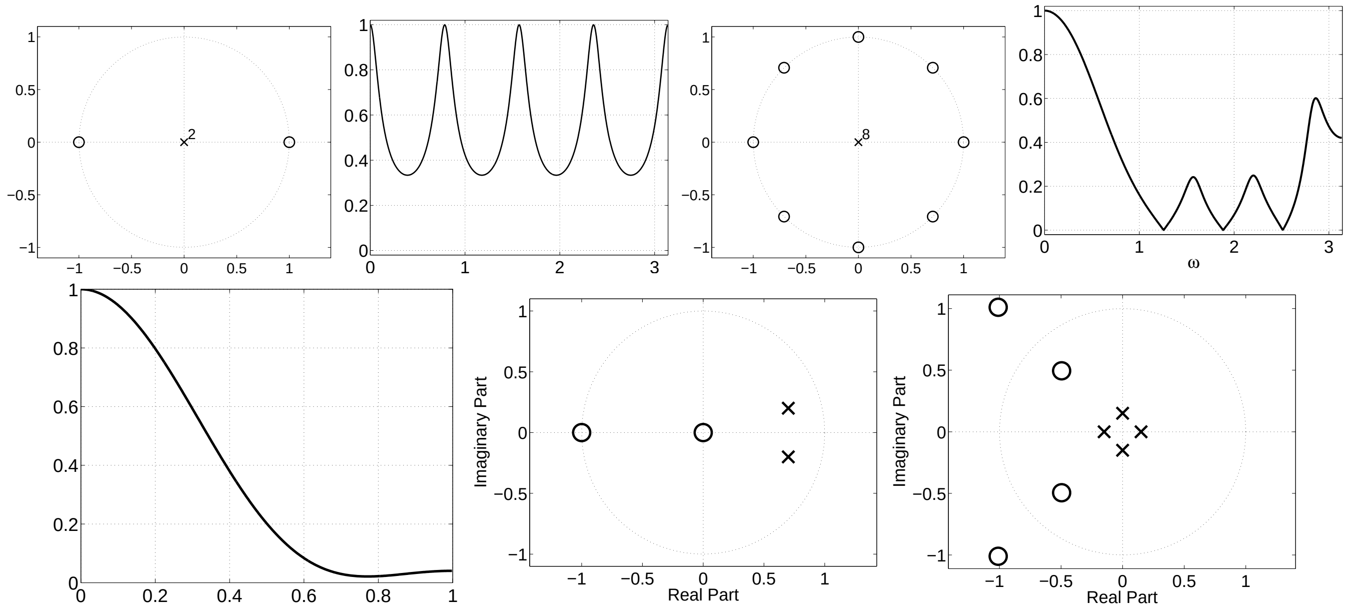

Figure 1: Figures of multichoice problem, top row (a), (b), (c), (d), bottom row: (e), (f), (g).

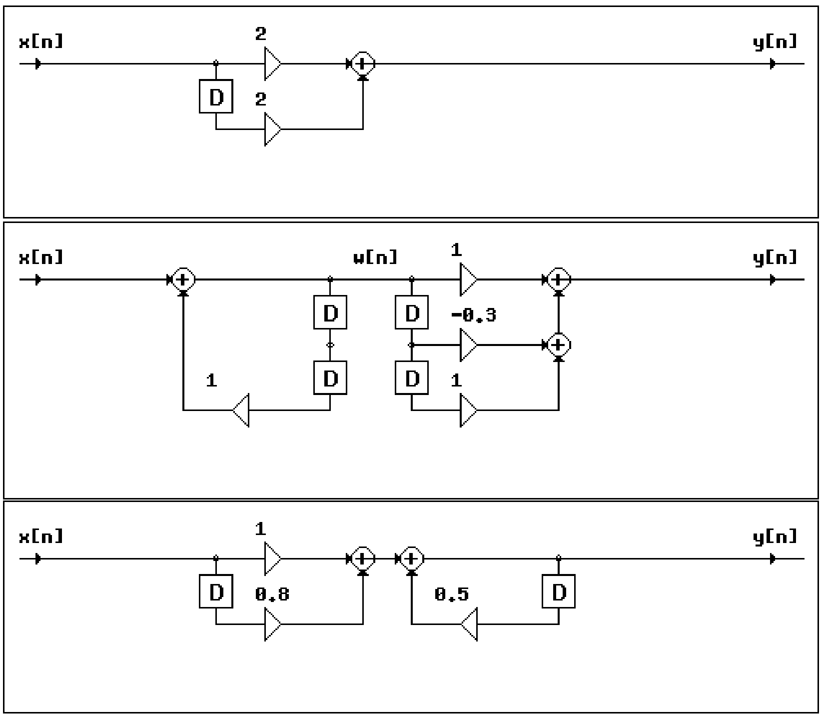

Figure 2: Figures of multichoice problem, (a), (b), (c).

2) (6 p) Real analog signal $x(t)$, whose spectrum $|X(j\Omega)|$ is drawn in Figure 3, is sampled with sampling frequency $f_s = 8000$ Hz into a sequence $x[n]$.

a) In the sampling process aliasing occurs. What would have been smallest sufficient sampling frequency, with which no aliasing would not happen?

b) Analog signal $x(t)$ is 0.2 seconds long. How many samples are there in the sequence $x[n]$?

c) Sketch the spectrum $|X(e^{j\omega})|$ of sampled sequence $x[n]$.

d) Sequence $x[n]$ is filtered with a LTI system, whose pole-zero plot is shown in Figure 3. After that filtered sequence $y[n]$ is reconstructed (ideally) to continuous-time $y_r(t)$. Skectch the spectrum $|Y_r(j\Omega)|$ in range $f = [0 \dots 20]$ kHz.

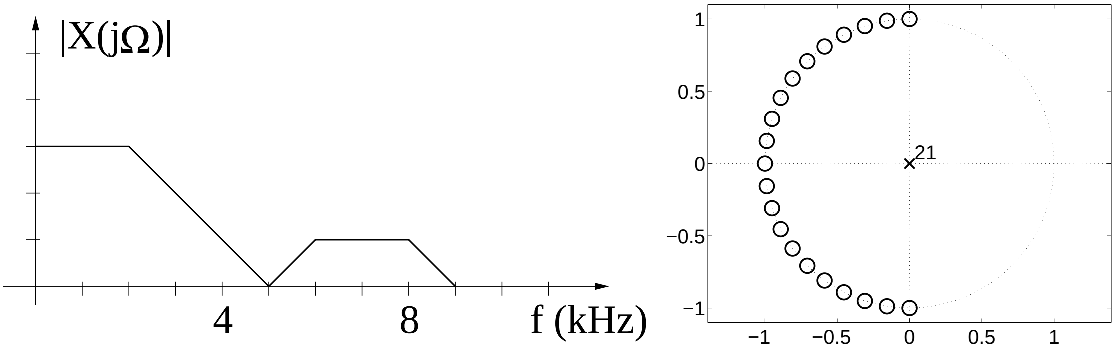

Figure 3: Spectrum left. Pole-zero plot right.

3) (6 p) Consider a system h, shown in Figure 4, which is constructed from four LTI subsystems h_1, h_2, h_3 and h_4. The following impulse responses are known

$$\begin{aligned} h_2[n] &= 3\delta[n] - \delta[n-1] + \delta[n-2] \\ h_3[n] &= -\delta[n] + \delta[n-1] + \delta[n-2] \\ h_4[n] &= 0.8^{n+1}\mu[n+1] \end{aligned}$$

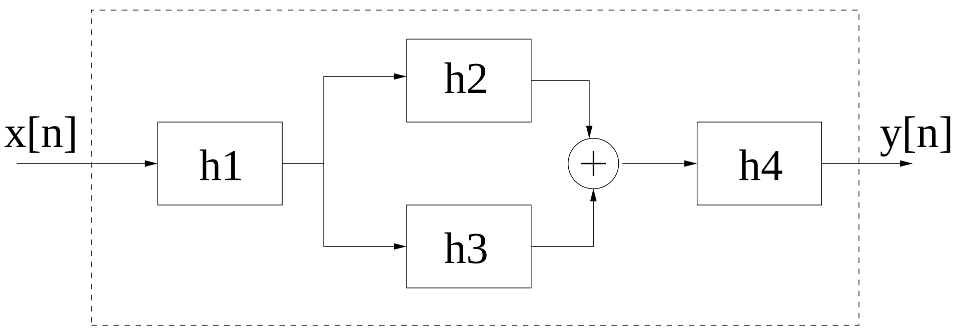

Figure 4: Filter h shown with LTI subsystems h_1, h_2, h_3 and h_4.

Determine impulse response $h_1[n]$ so that filter h is causal and symmetric bandstop filter with maximum amplification scaled to one. What is then the impulse response $h[n]$ (in close form, $h[n]$ as a function of n)? Compute separately values $h[0]$, $h[1]$, $h[2]$. Show clear intermediate steps.