T-61.3010 Digital Signal Processing and Filtering

Summer exam, Mon 5.6.2006 17-20. /Simula, Parviainen

You are not allowed to use any math table books of your own. A table of formulas is delivered. A (graphical) calculator is allowed.

Start a new task from a **new page**. Write all **intermediate steps**.

1) (7 x 1p = 0-6 p) Multichoice. There are 1-4 correct answers in the statements, but choose **one and only one.** Correct answer +1 p, wrong answer −0.5 p, no answer 0 p. You do not need to write why you chose your option. Reply to as many as you want. The maximum number of points is 6, and the minimum 0.

1.1 The fundamental period N_0 of the sequence $x[n] = \sin(0.25\pi n^2 + 0.25)$ is

(A) $N_0 = 2$
(B) $N_0 = 4$
(C) $N_0 = 8$
(D) $N_0 = 16$

1.2 The frequency response of a LTI system is $H(e^{j\omega}) = 2e^{-j3\omega} - e^{-j\omega} - e^{j\omega} + 2e^{j3\omega}$. Now

(A) the frequency response is (almost always) complex-valued
(B) the frequency response is always real and positive
(C) the phase response is linear
(D) the group delay is zero

1.3 Sequences $x[n]$ and $h[n]$ are convolved and z-transform is taken from the output

$$
\begin{aligned}
& Z\{x[n] \circledast h[n]\} \\
= & \sum_{n=-\infty}^{\infty} \Big(\sum_{k=-\infty}^{\infty} x[k]h[n-k] \Big) z^{-n} \\
= & \sum_{k=-\infty}^{\infty} x[k] \sum_{n=-\infty}^{\infty} h[n-k]z^{-n} \\
= & \sum_{k=-\infty}^{\infty} x[k] \sum_{m=-\infty}^{\infty} h[m]z^{-m}z^{-k} \\
= & \Big(\sum_{k=-\infty}^{\infty} x[k]z^{-k} \Big) \cdot \Big(\sum_{m=-\infty}^{\infty} h[m]z^{-m} \Big)
\end{aligned}
$$

What can be said about expressions above?

(A) It has been shown that convolution in frequency domain corresponds multiplication in time domain
(B) It has been shown that convolution in time domain corresponds multiplication in frequency domain
(C) It is not allowed to do the variable change $m = n - k$
(D) There are other errors than one in (C)

1.4 The filter in Figure 1(a):

(A) the transfer function is $H(z) = \frac{0.8+1.2z^{-1}+z^{-2}}{1-1.2z^{-1}-0.8z^{-2}}$
(B) it is allpass filter
(C) it is canonic with respect to delays
(D) it is FIR type

1.5 Analog filter $H(s) = 1/(s+0.5)$ is converted to a digital $H(z)$ using bilinear transform. The digital filter is

(A) $H(z) = 1/(z^{-1} + 0.5)$
(B) $H(z) = 2/(1 + 2z^{-1})$
(C) $H(z) = (2/3) \cdot (1 + z^{-1})/(1 - (1/3)z^{-1})$
(D) $H(z) = (1 + z^{-1}) \cdot (1 - z^{-1})/(1 + 0.5z^{-1})$

1.6 In order to avoid aliasing in the sampling process, the sampling period T_s has to be

(A) the same as the fundamental period T_0 of that component which has the longest fundamental period
(B) at least two times as long as the fundamental period T_0 of the highest frequency component
(C) at least half times larger than the highest frequency component f_h in the signal
(D) at most half of the fundamental period T_0 of the highest frequency component

1.7 The spectrum $|X(e^{j\omega})|$ of a real digital signal is shown in Figure 1(b). The spectrum is bandlimited in range $\pi/3 \leq \omega \leq 2\pi/3$. The sampling frequency is increased by factor $L = 3$, i.e., the signal is upsampled. After this there are frequency components in the range $0 \leq \omega \leq \pi$:

(A) nowhere
(B) everywhere
(C) at $\pi/3 \leq \omega \leq 2\pi/3$
(D) at $\pi/9 \leq \omega \leq 2\pi/9$, $4\pi/9 \leq \omega \leq 5\pi/9$, and $7\pi/9 \leq \omega \leq 8\pi/9$

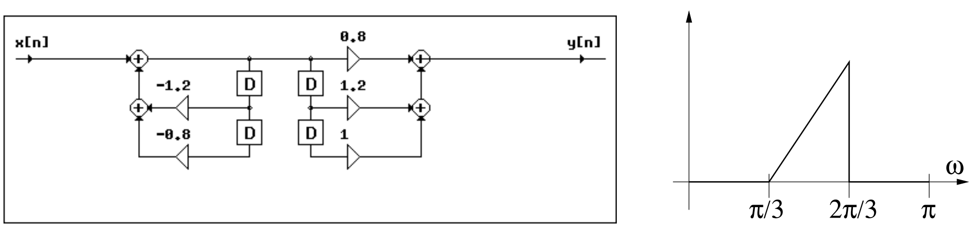

Figure 1: (a) and (b): figures for multichoise statements 1.4 and 1.7.

2) (6p) Input sequence $x[n]$ is fed into second order causal FIR filter (with empty registers), and the result is output $y[n]$. The first values of sequences are drawn in Figure 2 and the values are:

$$\begin{aligned} x[n] &= \{\underline{0.2068}, 0.7760, 0.9362, 0.6680, -0.1754, -0.7780, -0.9900, \ldots\} \\ y[n] &= \{\underline{0.1034}, 0.3880, 0.5715, 0.7220, 0.3804, -0.0550, -0.5827, \ldots\} \end{aligned}$$

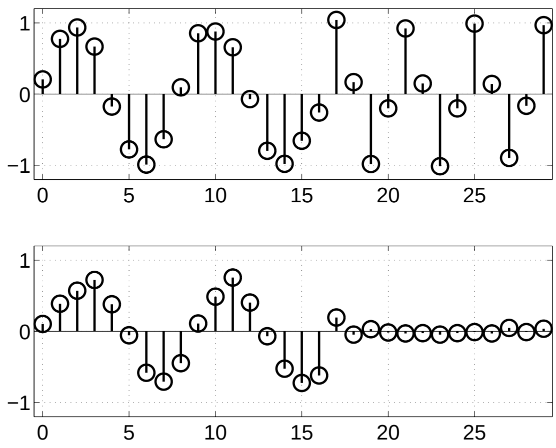

Figure 2: Problem 2, input $x[n]$ and output $y[n]$.

a) What is the impulse response of the filter $h[n]$?

b) What is the transfer function $H(z)$ of the filter?

c) Sketch the pole-zero diagram and the magnitude response.

d) Explain why the output $y[n]$ seems to be almost zero when $n \geq 18$.

3) (6p) Consider the filter in Figure 3.

a) Determine the transfer function $H(z)$.

b) Draw the pole-zero diagram. Compute the distances of zeros and poles from the origo.

c) Sketch the magnitude response $|H(e^{j\omega})|$, when $K = 1$. What is the type of the filter (notice a certain symmetry in coefficiens): lowpass / highpass / bandpass / bandstop / allpass?

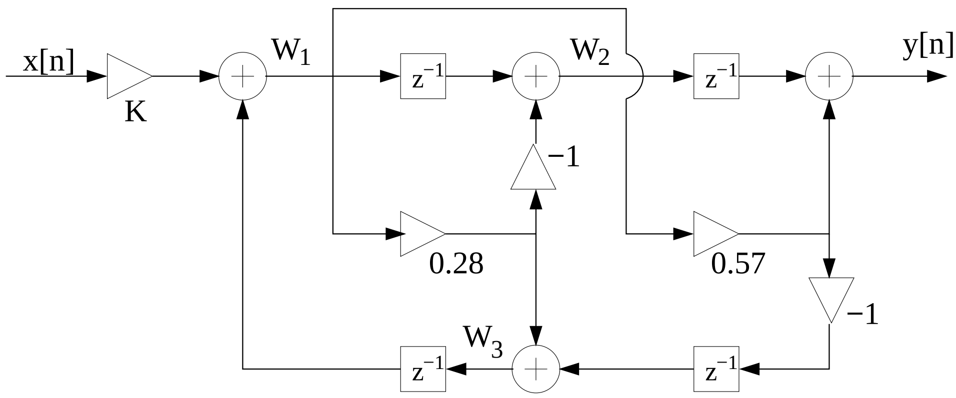

Figure 3: Filter in Problem 3.

4) (6p) Design a FIR filter with window method, when the cut-off frequency of the lowpass filter is at $f_c = 4000$ Hz and the sampling frequency is $f_T = 10000$ Hz. Window functions are represented in Table 1.

a) Sketch the frequency response of the ideal $H_{ideal}(f)$.

b) Compute the impulse response $h_{ideal}[n]$. of the corresponding ideal filter. Give the values, when $n = -2 \ldots 2$.

c) Compute the coefficients of the FIR filter $h_{FIR}[n]$ using window method and Hamming window $w_H[n]$, whose length is 5 ($M = 2$).

d) Examine the usefulness of this FIR filter, when in stopband 54.5 decibel minimum attenuation is required.

5) (6p) See the filter in Figure 4. The input values are represented with B bits. After multiplications the number of bits is $2B$. In order to get the number of bits in output to B, it is necessary to quantize values of $w[n]$ (block Q).

Quantization error can be compensated using so called error feedback (or error-shaping filter). In Figure 4 there is a second order filter with a second order error feedback system.

Write down first the difference equations for $e[n]$ and $w[n]$, and write down then in frequency domain the quantized output $Y(z)$ using input $X(z)$, filter modifing input $H_x(z)$, quantization noise $E(z)$, and filter modifing noise $H_e(z)$ in form of

$$Y(z) = H_x(z)X(z) + H_e(z)E(z)$$

and reply

a) how does the filter behave, if it is possible to use infinite wordlength, i.e. there is no quantization and $e[n] \equiv 0, \forall n$?

b) how does the spectrum of the total noise $E_{tot}(z) = H_e(z)E(z)$ look like if there is no compensation, i.e. $k = 0$, and if $e[n]$ is white noise so that $E(z) = 1$ for all frequencies?

c) with which simple value of k the effect of noise is suppressed in the passband?

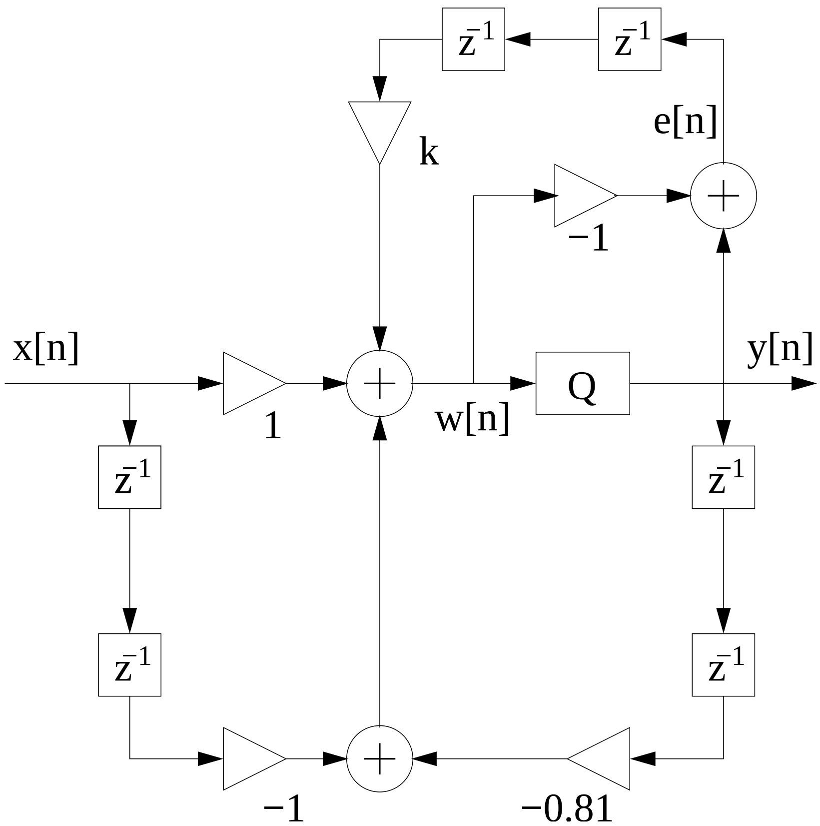

Figure 4: Second order system with second order error feedback.

Window	$w[n]$, $-M \leq n \leq M$	Length of main lobe Δ_{ML}	Relative side lobe A_{sl}	Mininum stopband attenu-ation	Length of transition band $\Delta\omega$
Rectangular	1	$4\pi/(2M+1)$	13.3 dB	20.9 dB	$0.92\pi/M$
Hann	$0.5 + 0.5\cos(\frac{2\pi n}{2M})$	$8\pi/(2M+1)$	31.5 dB	43.9 dB	$3.11\pi/M$
Hamming	$0.54 + 0.46\cos(\frac{2\pi n}{2M})$	$8\pi/(2M+1)$	42.7 dB	54.5 dB	$3.32\pi/M$
Blackman	$0.42 + 0.5\cos(\frac{2\pi n}{2M}) + 0.08\cos(\frac{4\pi n}{2M})$	$12\pi/(2M+1)$	58.1 dB	75.3 dB	$5.56\pi/M$

Table 1: Properties of window functions.