T-61.246 Digital Signal Processing and Filtering

2nd mid term exam / final exam 9th Dec 2004 at 16-19. Halls M, G, K.
If you are doing 2nd MTE, reply to problems 3, 4, 5, 6.
If you are doing final exam, reply to problems 1, 2, 4, 5, 6.
Write down, if you are doing 2nd MTE or final exam.
You may use a (graphical) calculator. You must clear all extra memory in your calculator. There is an additional formulae table given in the exam, but you can also use a math reference book of your own. **Write down all necessary steps which lead to the results.**

CS-department is collecting **course feedback** from all courses in autumn 2004.
PLEASE, GIVE FEEDBACK IN WEB
`http://www.cs.hut.fi/Opinnot/Palaute/kurssipalaute-en.html`.
The link can be found also from the course web page.

1. (6p, final exam)

   a) (2p) What is the fundamental period N_0 of the sequence $x[n] = 3\cos((\pi/4)n) + \sin((\pi/6)n - \pi/4)$?

   b) (2p) What is the convolution $y[n] = h[n] \circledast x[n]$ of the sequences $h[n] = \delta[n-1] - \delta[n-2]$ and $x[n] = 2\delta[n-1] + \delta[n-2] - \delta[n-3]$?

   c) (2p) Consider a causal and stable LTI filter with transfer function

   $$H(z) = \frac{1 - z^{-2}}{1 + 0.7z^{-1}}$$

   Derive the inverse transform $h[n]$ (impulse response)? Compute also $h[5]$.

2. (6p, final exam) Consider a sequence $x[n]$ in Figure **??**. It is a sampled version of a continuous signal $x(t)$, which is sampled using the sampling frequency of $f_T = 10$ kHz. X-axis contains the index number of n (not seconds). Sequence $x[n]$ is of form $x[n] = A\cos(2\pi(f/f_T)n + \theta)$.

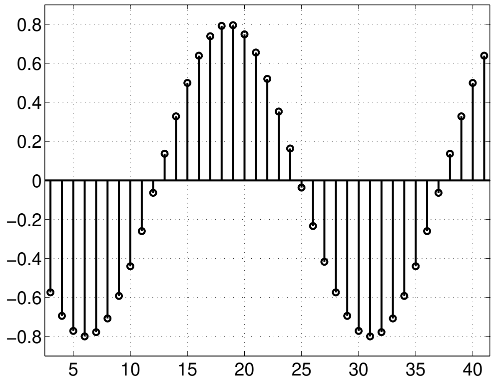

Figure 1: Sequence $x[n]$ of Problem 2.

   a) What is the period T_s between each sample in seconds?

   b) Determine the frequency of the sinusoidal and sketch the spectrum $X(e^{j\omega})$ of a discrete-time sequence $x[n]$ in range $0 \ldots f_T$ Hz.

   c) Continuous-time signal $\hat{x}(t)$ is reconstructed from the sequence $x[n]$. Sketch the spectrum $|\hat{X}(j\Omega)|$ in range $0 \ldots f_T$ Hz.

   d) What can be said about the original signal $x(t)$, from which the sequence $x[n]$ is a sampled version?

3. (6p, MTE2) Are the following statements true (T) or false (F)? A right answer gives +1p, no answer 0 p, and a wrong answer –0.5p. Answer to as many statements as you want. You do not need to explain. The total amount of points is 0-6p.

   a) One possible polyphase realization of a FIR filter $H(z) = 1 - 0.3z^{-1} - 0.3z^{-2} + z^{-3}$ is $H(z) = E_0(z^2) + E_1(z^2)$, where $E_0(z) = 1 - 0.3z^{-1}$ and $E_1(z) = -0.3 + z^{-1}$.

   b) In the impulse-invariant method the values of the impulse response $h[n]$ are directly the coefficients of the analog filter $H(s)$.

   c) Scaling of the filter is used to suppress the signal in order to reject overflows, and at the same time signal-to-noise ratio (SNR) is improved.

   d) Matlab code `freqz(B, A)` produces a curve in Figure 2(a), when `B` and `A` are computed corrected beforehand, and the sampling frequency is 16 kHz.

   e) Matlab code `zplane(B, A)` produces a curve in Figure 2(b), when `B` and `A` are computed corrected beforehand, and the sampling frequency is 16 kHz.

   f) The order of the filter in Figure 2(c) is 4.

   g) Downsample a cosine sequence of 1000 Hertz so that the original sampling frequency 16 kHz is dropped to 1/4 of the original, that is with factor $M = 4$. Statement: The frequency of the downsampled signal is 250 Hz.

   h) A cascade (series) system of second-order systems is more sensitive to quantization of coefficients than the corresponding direct form structure.

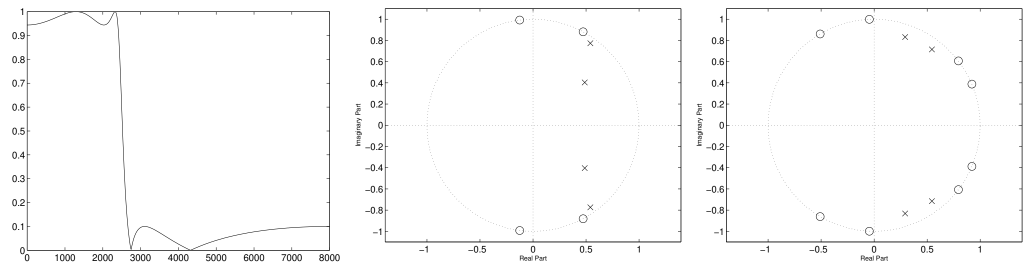

Figure 2: Figure (a), (b), and (c) for Problem 3.

4. (6p, MTE2, final exam) Consider a second order LTI system, whose transfer function is

$$H_1(z) = \frac{1 - 1.18z^{-1} + z^{-2}}{1 + 1.58z^{-1} + 0.64z^{-2}}$$

where poles are at $p = -0.79 \pm 0.1261j$ and zeros at $z = 0.59 \pm 0.8074j$, and the maximum of the amplitude response is at $\omega = \pi$.

   a) (4p) Using the filter $H_1(z)$ implement a fourth-order bandpass filter $H_2(z)$, whose maximum is at $\omega = \pi/2$. Give the filter $H_2(z)$ in format

$$H_2(z) = K \cdot \frac{1 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3} + b_4 z^{-4}}{1 + a_1 z^{-1} + a_2 z^{-2} + a_3 z^{-3} + a_4 z^{-4}}$$

   b) (2p) Determine the coefficient K so that the maximum value of the bandpass filter is scaled to one.

5. (6p, MTE2, final exam) Design a FIR filter with window method, when the cut-off frequency of the lowpass filter is at $f_c = 3000$ Hz and the sampling frequency is $f_T = 12000$ Hz. Window functions are represented in Table 1.

   a) (1p) Sketch the frequency response of the ideal $H_{ideal}(f)$.

   b) (2p) Compute the impulse response $h_{ideal}[n]$. of the corresponding ideal filter. Give the values, when $n = -3 \ldots 3$.

   c) (2p) Compute the coefficients of the FIR filter $h_{FIR}[n]$ using window method and Hann window $w_H[n]$, whose length is 7 ($M = 3$).

   d) (1p) Examine the usefulness of this FIR filter, when in stopband 43,9 decibel minimum attenuation is required.

Window	$w[n]$, $-M \leq n \leq M$	Length of main lobe Δ_{ML}	Relative side lobe A_{sl}	Mininum stopband attenu-ation	Length of transition band $\Delta\omega$
Rectangular	1	$4\pi/(2M+1)$	13.3 dB	20.9 dB	$0.92\pi/M$
Hann	$0.5 + 0.5\cos(\frac{2\pi n}{2M})$	$8\pi/(2M+1)$	31.5 dB	43.9 dB	$3.11\pi/M$
Hamming	$0.54 + 0.46\cos(\frac{2\pi n}{2M})$	$8\pi/(2M+1)$	42.7 dB	54.5 dB	$3.32\pi/M$
Blackman	$0.42 + 0.5\cos(\frac{2\pi n}{2M}) + 0.08\cos(\frac{4\pi n}{2M})$	$12\pi/(2M+1)$	58.1 dB	75.3 dB	$5.56\pi/M$

Table 1: Properties of window functions.

6. (6p, MTE2, final exam) **Choose either A or B.**

6A. Essay: FFT-algorithms, especially "Decimation-in-Time" and "Decimation-in-Frequency". You do not have to derive formulas.

6B. See the filter below. The input values are represented with B bits. After multiplications the number of bits is $2B$. In order to get the number of bits in output to B, it is necessary to quantize values of $w[n]$ (block Q).

Quantization error can be compensated using so called error feedback (or error-shaping filter). In Figure 3 there is a second order filter with a first order error feedback system.

Write down first the difference equations for $e[n]$ and $w[n]$, and write down then in frequency domain the quantized output $Y(z)$ using input $X(z)$ and quantization noise $E(z)$, and reply

a) how does the filter behave, if it is possible to use infinite wordlength, i.e. there is no quantization and $e[n] \equiv 0, \forall n$?

b) how does the spectrum of the noise look like if there is no compensation, i.e. $k = 0$, and if $e[n]$ is white noise so that $E(z) = 1$ for all frequencies?

c) with which simple value of k the effect of noise is suppressed in the passband?

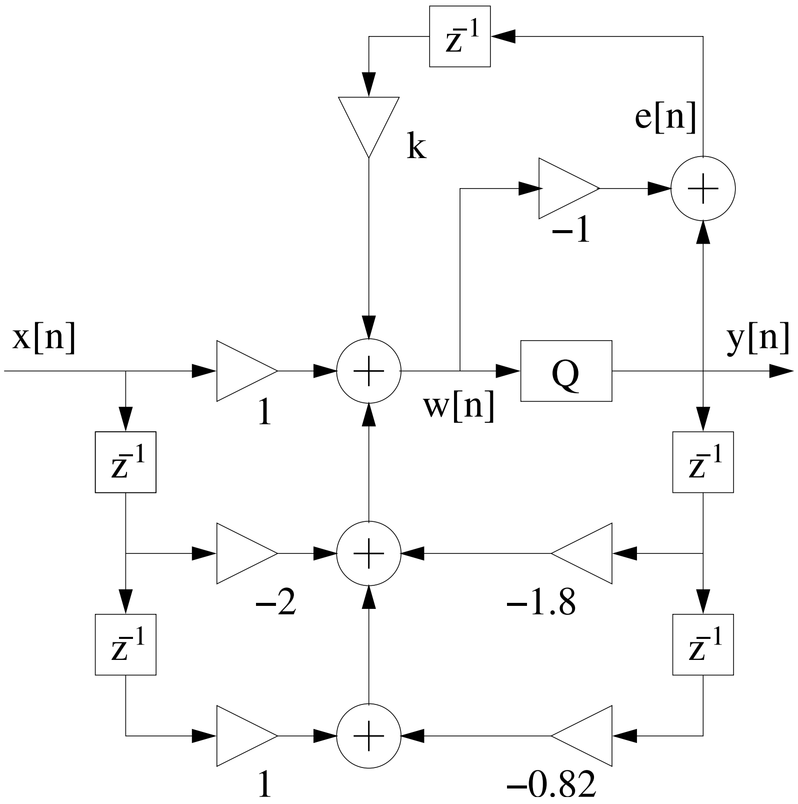

Figure 3: Second order system with first order error feedback.