T-61.246 Digital Signal Processing and Filtering

UPDATED PAPER. 1st mid term exam, Mon 18th Oct 2004 at 16-19 in hall A.

You are not allowed to use any calculator nor math formula book. All papers except this one has to be returned. A table of formula is delivered.
Start a new problem from a new page. Write down the intermediate steps, too.

1) (6p) What is the (mathematical) definition for periodic signals?

   It is known that the signal $x(t) = \cos(\pi t/3 + \pi/2) + 2\cos(2\pi t^2/16)$ **is not** periodic. Show using the definition that the discrete-time sequence

$$x[n] = \cos(\pi n/3 + \pi/2) + 2\cos(2\pi n^2/16)$$

   is periodic, and find the fundamental period N_0.

2) (6p) Examine the following sequences, where $x[n]$ depicts the input for the LTI system, and $h[n]$ is the impulse response of the system:

$$\begin{aligned} x[n] &= 3\delta[n-1] - 2\delta[n-2] + \delta[n-3] \\ h[n] &= (-1)^{n-3}\mu[n+3] \end{aligned}$$

   a) (2p) Draw the sequences $x[n]$ and $h[n]$.
   b) (1p) Is the LTI filter stable? Explain.
   c) (1p) Is the LTI filter causal? Explain.
   d) (2p) The output sequence from the system is given by $y[n] = h[n] \circledast x[n]$. Compute $y[2004]$.

3) (9p) Examine a discrete-time system, whose transfer function is

$$H(z) = \frac{1 - 0.64z^{-2}}{1 - 1.4z^{-1} + 0.98z^{-2}}$$

   a) (1p) Sketch the pole-zero plot with the unit circle.
   b) (1.5p) Sketch the amplitude response. Is the filter lowpass / highpass / bandpass / bandstop / all-pass?
   c) (1.5p) What is the difference equation of the system?
   d) (1p) Is the filter stable? Explain.
   e) (1p) Draw the flow/block diagram of the filter.
   f) (1p) Is the filter FIR or IIR? Explain.
   g) (1p) What are the values for the impulse response $h[n]$, when $n = 0 \dots 3$?
   h) (1p) Why it can be said that the system is LTI?

4) (3p) From a pole-zero plot/diagram the amplitude response can be roughly estimated.

Connect a pole-zero plot to a corresponding amplitude response. There is one pole-zero plot, which does not fit to one amplitude response. Write down the three correct pairs (LETTER, number).

In the left column there are the pole-zero plots A..D, in the right column the amplitude responses 1..4. The x-axis in amplitude responses is $0 \ldots 1$ corresponding the normalized angular frequency $0 \ldots \pi$ (frequencies $0 \ldots f_s/2$).