T-61.3010 Digital Signal Processing and Filtering

Mid term exam 2 / Final exam. Wed 13.5.2009 16-19. Hall A.

You can do MTE2 only once either 6.5. or 13.5. Mid term exam: Problems 1 and 2.

You can do final exam only once either 6.5. or 13.5. Final exam: Problems 3, 4, 5, 6, and 7. Begin each problem from a new page.

You can have a function calculator. If not, apply variables like $k = \sin(0.2\pi)$. You are not allowed to have any math formula book of your own. You will be given a course formula paper.

1) (12 x 1p, 0-12 p, **ONLY MTE2**) Multichoice. There are 1–4 correct answers, but choose **one and only one.** Fill in **into a separate form**, which will be read optically.

Correct answer +1 p, incorrect −0.5 p, no answer 0 p. You do not need to explain your choices. Reply to as many statements as you want. The maximum points of this problem is 12 and the minimum 0.

1.1 Consider a block diagram in Figure 1, where three subsystems $S(z)$, $R(z)$ and $T(z)$ are LTI. The transfer function of the filter is

(A) $H(z) = \frac{R(z) \cdot T(z)}{1 - R(z) \cdot S(z)}$

(B) $H(z) = \frac{R(z) \cdot T(z)}{S(z)}$

(C) $H(z) = R(z) \cdot S(z) \cdot T(z)$
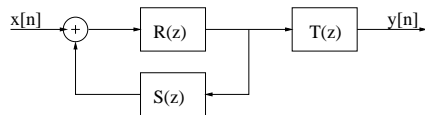
(D) $H(z) = (1 + S(z)) \cdot R(z) \cdot T(z)$

Figure 1: Multichoice 1.1.

1.2 Consider causal and stable LTI filter in Figure 2(a)

(A) Transfer function of filter is constant $H(z) = \frac{1}{2}$.

(B) Pole-zero plot of filter is in Figure 2(b).

(C) Impulse response of filter is $0.5h[n] = \delta[n] - 0.2\delta[n-1] - 0.5\delta[n-2] + 0.2h[n-1] + 0.5h[n-2]$.

(D) Filter is drawn as a canonic (with respect to delays) direct form II structure.
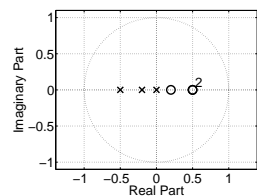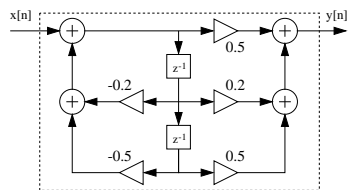

Figure 2: Multichoice 1.2: block diagram and the pole-zero plot of the option (**B**) .
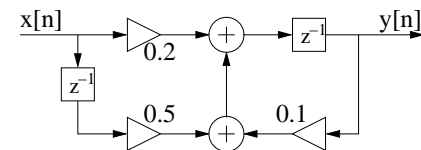
1.3 Consider a LTI system in Figure 3.

Figure 3: Multichoice 1.3, LTI filter.

(A) The system is expressed as a poly-phase structure FIR filter.

(B) Difference equation of the filter is $y[n] = 10y[n-1] - 2x[n] - 5x[n-1]$.

(C) The equivalent transpose structure of the filter is shown in Figure 4.

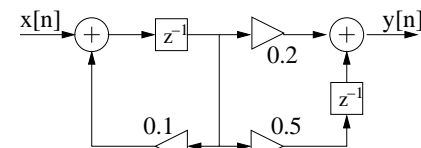(D) There is a feedback without any delay ("delay free loop"), and therefore it cannot be realized.

Figure 4: Multichoice 1.3, option (**C**) .

1.4 The transfer function $H(z)$ of a high pass filter with monotonic passband is $H(z) = K \cdot (1 - z^{-1} + z^{-2} - z^{-3})$. If the maximum of the filter is scaled to one, what is the gain K?

(A) $K = 0.25$.

(B) $K = 0.5$.

(C) $K = 1$.

(D) $K = \infty$.

1.5 In the window method of the digital FIR filter design, the purpose of the window function $w[n]$ itself is

(A) to map left plane of s-plane into inside of the unit circle in z-plane.

(B) to cut the infite-length impulse response $h_{\text{ideal}}[n]$ to finite length.

(C) to attenuate frequency components over half of the sampling frequency.

(D) to compute energy of impulse response $h_{\text{ideal}}[n]$ in order to ensure that the filter is stable.

1.6 First in digital IIR filter design the specifications are written and after that the order N of the filter is estimated.

(A) Choosing the order does not have any effect on the computation of the filter as long as the specifications are fulfilled.

(B) Order is chosen to be as low as possible as long as the specifications are fulfilled.

(C) Order N is smaller and smaller when the cut-off frequency ω_c is smaller and smaller.

(D) Digital IIR filter of order N is of form $H(z) = b_0 + b_1 z^{-1} + \ldots + b_N z^{N-1}$

1.7 Analog filter $H(s) = \Omega/(s + \Omega)$, where prewarped cut-off frequency $\Omega = k \cdot 0.2$, is converted to digital $H(z)$ using bilinear transform. The digital filter is

(A) $H(z) = (1/3) \cdot \frac{1}{1 + 0.5z^{-1}}$

(B) $H(z) = (2/3) \cdot \frac{1 + z^{-1}}{1 + (1/3)z^{-1}}$

(C) $H(z) = 2 \cdot \frac{1 + z^{-1}}{1 - 0.5z^{-1}}$

(D) $H(z) = 2 \cdot [1 + z^{-1}]$

1.8 The following equations have been derived from the block diagram of a digital filter in Figure 5

$$y[n] = ABw[n] + Be[n]$$
$$w[n] = x[n] + y[n]$$

where A and B are positive constants.

(**A**) The transfer function with respect to x and y is $H(z) = \frac{AB+B}{1-AB}$.

(**B**) The impulse response with respect to x and y is $h[n] = (AB)^n$.

(**C**) The filter cannot be physically realized because there are two inputs.

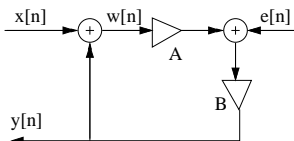(**D**) The filter cannot be physically realized because there is no delay.

Figure 5: Multichoice 1.7.

1.9 Effects of finite wordlength are related especially to

(**A**) the use of impulse invariant method in IIR filter design.

(**B**) mathematical non-periodic behaviour of the sequence, when the period N_0 is not mathematically integer even if in practice it would be ($10\pi \approx 31$).

(**C**) the computation of the spectrogram of an audio signal.

(**D**) the quantization of the filter coefficients.

1.10 Periodic sequence ($N_0 = 4$) $x[n] = \{\ldots, \underline{2}, 0, 0, 9, \ldots\}$ is fetched into a digital system

$x[n] \rightarrow \boxed{\downarrow 5} \rightarrow \boxed{\uparrow 2} \rightarrow y[n]$.

(**A**) $y[n]$ is a periodic sequence ($N_0 = 4$) $y[n] = \{\ldots, 0.8, 0, 0, 3.6, \ldots\}$.

(**B**) $y[n]$ is a periodic sequence with fundamental period $N_0 = 8$.

(**C**) $y[n]$ is a periodic sequence with fundamental period $N_0 = 10$.

(**D**) $y[n] = x[n]$.

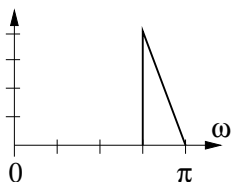1.11 Examine the spectrum $|X(e^{j\omega})|$ of a real sequence $x[n]$ in Figure 6.

Figure 6: Multichoice 1.11 tutkittava $|X(e^{j\omega})|$.

The sequence is fed into a multirate system $x[n] \rightarrow \boxed{\downarrow 4} \rightarrow y[n]$.

What can be said about the spectrum of the output (assumption: scales in y-axis correct)?

(**A**) Spectrum of the output $y[n]$ is in Figure 7(a).

(**B**) Spectrum of the output $y[n]$ is in Figure 7(b).

(**C**) Spectrum of the output $y[n]$ is in Figure 7(c).
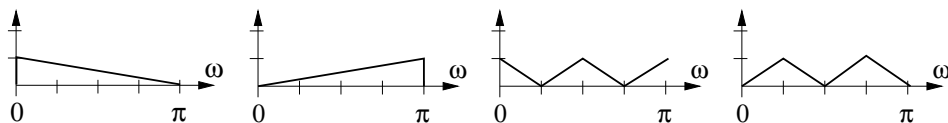
(**D**) Spectrum of the output $y[n]$ is in Figure 7(d).

Figure 7: Multichoice 1.11, options (**A**) , (**B**) , (**C**) ja (**D**) .

1.12 What for the following lines of code are suited best?

```
[x, fT] = wavread('mySignal.wav');
y    = zeros(size(x));
wL   = 256;
m    = 0;
Z    = [zeros(17,1); ones(223,1); zeros(16,1)];
for k = 1 : wL : length(x)-wL
    m    = m + 1;
    tmpx = x(k : k+wL-1);
    tmpxF = fft(tmpx);
    tmpyF = tmpxF .* Z;
    y(k : k+wL-1) = real(ifft(tmpyF));
end;
```

(**A**) To compute the signal energy in small time frames of 256 samples.

(**B**) To create an "echo effect".

(**C**) To change the sampling frequency to the fraction of 17/256.

(**D**) To filter the signal with a highpass filter with the normalized cut-off angular frequency $\omega_c \approx \pi/8$.

2) (6p, **MID TERM EXAM**) Choose either 2A or 2B. Write down an exam essay, where introduction, body of essay, and summary. Use clear visualisations if needed.

2A) **OPTION A.** FFT algorithms. In addition to common text you can use as an example "radix-2 DIT FFT" algorithm whose butterfly equations and W_N are given in the formula table. Compute with intermediate steps FFT for a sequence ($N = 4$) $x[n] = -1\delta[n] - 4\delta[n-1] + 2\delta[n-2] + 3\delta[n-3]$.

2B) **OPTION B.** Automatic speech recognition.

3) (6p, **ONLY FINAL EXAM**)

a) Explain that the digital system $y[n] = x_1[n] \cdot (2 \cdot x_2[n-2])$ is not a LTI system.

b) Show that the digital LTI system with impulse response $h[n] = (-0.5)^{n+2} \mu[-n-2]$, is not causal nor stable.

c) Find the fundamental period N_0 of the whole sequence $x[n]$

$$x[n] = 2\cos(2\pi \cdot (500/12000)n - \pi/6) + 3\cos(2\pi \cdot (3200/12000)n + \pi/8)$$

4) (6p, **ONLY FINAL EXAM**) Consider a LTI system which is composed of four simple filters

$$
\begin{aligned}
h_1[n] &= \delta[n] + 0.5\delta[n-1] \\
h_2[n] &= 0.5\delta[n] - 0.25\delta[n-1] \\
h_3[n] &= 2\delta[n] \\
h_4[n] &= -2 \cdot 0.5^n \mu[n]
\end{aligned}
$$

as shown in Figure 8.

a) Write down the form of the impulse response $h[n]$ of the whole system using impulse responses $h_1[n]$, $h_2[n]$, $h_3[n]$, and $h_4[n]$. Equivalently, write down the transfer function $H(z)$ of system using transfer functions of the subsystems.

b) Compute $h[n]$ and $H(z)$.

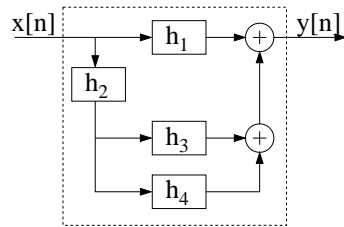c) Is the filter lowpass / highpass / bandpass / bandstop / allpass?

Figure 8: Problem 4, LTI system $h[n]$ with four LTI subsystems.

5) (6p, **ONLY FINAL EXAM**) Consider a LTI system in Figure 9.

a) Determine the difference equations of the system and after that the transfer function in the format

$$H(z) = G \cdot \frac{1 + b_1 z^{-1} + \ldots + b_M z^{-M}}{1 + a_1 z^{-1} + \ldots + a_N z^{-N}}$$

b) The structure of the filter is canonic with respect to delays, if the number of the delays is the same as the order of the filter. Is the filter in the figure canonic wrt delays?

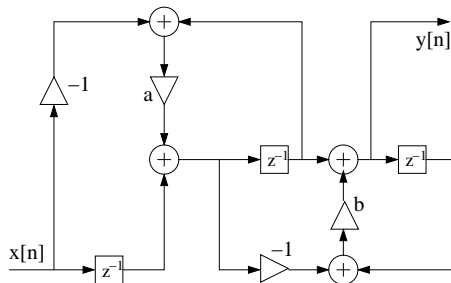c) What values coefficients a and b may get so that the filter is stable?

Figure 9: Problem 5, flow diagram of LTI filter.

6) (6p, **ONLY FINAL EXAM**) Consider a digital signal, whose spectrum is depicted in Figure 10. We are interested in the band up to 5 kHz. Noise begins from 6 kHz. The sampling frequency is $f_T = 48$ kHz.

Design a digital FIR-filter using Window method ("Truncated Fourier series") and Hamming window.

Hamming window and some parameters are given in the table below when the filter coefficients are computed in range of $-M \leq n \leq +M$.

Window	$w[n]$, $-M \leq n \leq M$	Min. stopband attenuation	Length of transition band $\Delta\omega$
Hamming	$0.54 + 0.46\cos((2\pi n)/(2M))$	54.5 dB	$3.32\pi/M$

a) Sketch the filter specifications as a diagram.

b) In the column "Length of transition band $\Delta\omega$" there is the width of the transition band (between passband and stopband) as normalized angular frequency, when M is given. Estimate the order N of your filter.

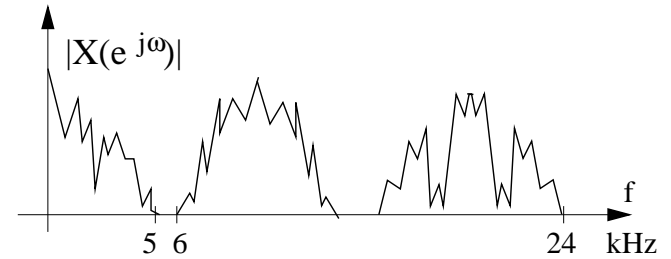c) Compute the following values of the impulse response $h[-M]$, $h[-3]$, $h[0]$, $h[3]$ ja $h[2009]$.

Figure 10: Problem 6, input spectrum $|X(e^{j\omega})|$.

7) (6p, **ONLY FINAL EXAM**) Consider a synthetic sequence

$$x[n] = 3\cos(2\pi \cdot (200/f_T)n) + 2\sin(2\pi \cdot (1200/f_T)n - \pi/6) + \cos(2\pi \cdot (3400/f_T)n + \pi/8)$$

with sampling frequency $f_T = 12000$ Hz.

a) Sketch the spectrum $|X(e^{j\omega})|$ in range $0 \ldots f_T/2$.

b) Sketch the spectrum $|Y_1(e^{j\omega})|$ of sequence $y_1[n]$ when $x[n]$ is downsampled with $M = 6$, that is,

$x[n] \rightarrow \boxed{\downarrow 6} \rightarrow y_1[n]$.

c) Sketch the spectrum $|Y_2(e^{j\omega})|$ of sequence $y_2[n]$ when $x[n]$ is first filtered with a lowpass decimation filter ("anti-aliasing filter") and after that downsampled with $M = 6$, that is

$x[n] \rightarrow \boxed{LP} \rightarrow \boxed{\downarrow 6} \rightarrow y_2[n]$.

Justify your choice of cut-off frequency in the decimation filter.