T-61.3010 Digital Signal Processing and Filtering

Mid term exam 2 / Final exam. Wed 14.5.2008 16-19. Hall A.

You can do MTE2 only once either 7.5. or 14.5. MTE2: Problems 1 and 2.

You can do final exam only once either 7.5. or 14.5. Final exam: Problems 2, 3, 4, 5, and 6. Begin each problem from a new page.

You can have a (graphical) function calculator (extra memory erased) but not any math formula book. You will be given a course formula paper and a multichoice sheet for Problem 1 (MTE2).

1) (10 x 1p, max 8 p, **ONLY MTE2**) Multichoice There are 1-4 correct answers, but choose **one and only one.** Fill in **into a separate form**, which will be read optically.

   Correct answer +1 p, incorrect −0.5 p, no answer 0 p. You do not need to explain your choices. Reply to as many statements as you want. The maximum points of this problem is 8 and the minimum 0.

   Statements 1.1 – 1.4 are core of this course and straigt forward. Statements 1.5 – 1.10 need probably some computation on paper.

   1.1 Consider causal and stable LTI filter in Figure 1(a)
   - **(A)** Transfer function of filter is constant $H(z) = \frac{1}{2}$
   - **(B)** Impulse response of filter is $0.5h[n] = \delta[n] - 0.2\delta[n-1] - 0.5\delta[n-2] + 0.2h[n-1] + 0.5h[n-2]$
   - **(C)** Pole-zero plot of filter is in Figure 1(b)
   - **(D)** Filter is drawn as a canonic (with respect to delays) direct form II structure

   1.2 First in digital IIR filter design the specifications are written and after that the order N of the filter is estimated.
   - **(A)** Order is always $N = 1$
   - **(B)** Order is chosen to be as high as possible in order to get effective attenuation in stopband
   - **(C)** Order is chosen to be as low as possible as long as the specifications are fulfilled
   - **(D)** Digital IIR filter of order N is of form $H(z) = b_0 + b_1 z^{-1} + \ldots + b_N z^{N-1}$

   1.3 Windowed Fourier series method (window method) in digital filter design:
   - **(A)** The impulse response of the filter to be designed is received as a product of an ideal impulse response and a window function
   - **(B)** It is one method for digital IIR filter design
   - **(C)** Window means a sequence $w[n]$, which is the ideal impulse response of the filter
   - **(D)** Typical window functions of the method are Butterworth, Chebychev, and elliptic

   1.4 Consider a sequence $x[n] = \{\underline{0.5},\ 0.3,\ 0.1,\ -0.2,\ -0.3,\ 0.1\}$, whose sampling period is $T = 1$ day. The sequence is downsampled by factor $M = 2$, after that we get $x_d[n]$.
   - **(A)** The new sampling frequency is $f_T = 44100$ Hz
   - **(B)** $x_d[n] = \{\underline{0.5},\ 0,\ 0.3,\ 0,\ 0.1,\ 0,\ -0.2,\ 0,\ -0.3,\ 0,\ 0.1\}$
   - **(C)** $x_d[n] = \{\underline{0.25},\ 0.15,\ 0.05,\ -0.1,\ -0.15,\ 0.05\}$
   - **(D)** $x_d[n] = \{\underline{0.5},\ 0.1,\ -0.3\}$

   1.5 The step response $s[n]$ of LTI system is received when the input is unit step function $\mu[n] = \{\ldots,\ 0,\ 0,\ \underline{1},\ 1,\ 1,\ \ldots\}$. What can be said about step response of LTI system in Figure 2(a)?
   - **(A)** Step response is $s[n] = \mu[n] - 2\mu[n-1] + \mu[n-2]$
   - **(B)** Value of the step response at $n = 14$ is $s[14] \approx -612000$ with three significant digits
   - **(C)** Step response $s[n]$ converges asymptotically to zero with large n
   - **(D)** Step response $s[n]$ diverges and grows without any bound with large n

   1.6 Consider specifications of digital FIR filter, see Figure 2(b). In passband we allow ripple between $0.9 \ldots 1.1$, whereas in stopband ripple must be between$-0.01 \ldots 0.01$, that is, the absolute value is less than 0.01. The normalized angular cut-off frequency of the lowpass filter is $\omega_p = 0.25\pi$, when the sampling frequency is $f_T = 10000$ Hz.

   We want to compare FIR filter to an IIR filter, whose specifications are given in x-axis with frequencies (Hz) and in y-axis with power decibel values: $\alpha_{max} \approx -20\log_{10}(1 - 2\delta_p)$ for passband maximum ripple and $\alpha_s = -20\log_{10}(\delta_s)$ for stopband minimum attenuation. When FIR specs are converted, we get, e.g.,
   - **(A)** $\alpha_{max} \approx 1.9$ dB and $\alpha_s \approx 20$ dB
   - **(B)** $\alpha_{max} \approx 3.0$ dB and $f_p = 2500$ Hz
   - **(C)** $\alpha_s \approx 40$ dB and $f_p = 1250$ Hz
   - **(D)** Filter order will be 2

1.7 Examine the second-order IIR filter with quantization block Q and first-order error-shaping in Figure 3(a). Let us write $w[n]$ and replace Q with noise source $e[n]$ as shown in Figure 3(b). Now you can write two difference equations, one $y[n] = \ldots$ and the other $w[n] = \ldots$.

Next, you can write down the output in frequency domain

$$Y(z) = H_x(z) \cdot X(z) + H_e(z) \cdot E(z)$$

where $H_x(z)$ is the actual filter and $H_e(z)$ is the filter for quantization error.

(A) These are $H_x(z) = \frac{1-1.8z^{-1}+0.82z^{-2}}{1+1.7z^{-1}+0.72z^{-2}}$ ja $H_e(z) = \frac{kz^{-1}}{1+1.7z^{-1}+0.72z^{-2}}$

(B) These are $H_x(z) = \frac{1+1.8z^{-1}+0.82z^{-2}}{1-1.7z^{-1}-0.72z^{-2}}$ ja $H_e(z) = \frac{1+kz^{-1}}{1-1.7z^{-1}-0.72z^{-2}}$

(C) These are $H_x(z) = \frac{1-1.8z^{-1}+0.82z^{-2}}{1+1.7z^{-1}+0.72z^{-2}}$ ja $H_e(z) = \frac{1+(k-1.8)z^{-1}+0.82z^{-2}}{1+1.7z^{-1}+0.72z^{-2}}$

(D) None of pairs above is not true.

1.8 Continue from 1.7. Suppose quantization error as white noise, whose spectrum $E(z) = 1$. What is the best value for k, so that total noise $E_{\mathrm{TOT}}(z)$ is shifted from interesting passband.

(B) $k = -1$

(A) $k = 0$

(C) $k = 1$

(D) $k = 1.8$

1.9 In Mitra's book it is explained and proved that the amplitude response $|H(e^{j\omega})|$ of Butterworth digital filter is monotonic (no ripple) through the whole band $(0,\ \pi)$. This means that the denominator polynomial of a lowpass filter of order N has N roots at the same $z = -1$.

First order Butterworth filter is computed and drawn with Matlab commands `[B, A] = butter(1, 0.25); zplane(B, A);` in Figure 4(a). Similarly, 21st order Butterworth is drawn in Figure 4(b) by commands `[B, A] = butter(21, 0.25); zplane(B, A);`. Consider now the pole-zero plot of the latter filter (kuvan 4(b)) where 21 zeros and around $z = -1$. This is contradiction between Matlab and the theory.

(A) Theory is wrong, because for each filter with order $N > 1$ zeros are situated on a circle with origin at $z = -1$ just as shown in the figure

(B) Mitra is wrong, because scientific articles after the publication of the book have proved that the amplitude response is not monotonic

(C) Matlab is wrong, because computing N-multiple root from a polynomia is not numerically accurate

(D) Matlab is wrong, because the command `zplane` is not able to draw multiple zeros

1.10 What can you do with a working piece of Matlab code below?

```
[x, fT] = wavread('mysignal.wav');
y    = zeros(size(x));
wL   = 256; m = 0;
Z    = [zeros(33,1); ones(191,1); zeros(32,1)];
for k = 1 : wL : length(x)-wL
   m      = m + 1;
   tmpx   = x(k : k+wL-1);
   tmpxF  = fft(tmpx);
   tmpyF  = tmpxF .* Z;
   y(k : k+wL-1) = real(ifft(tmpyF));
end;
```

(A) In `for` loop one changes the sampling frequency of the signal

(B) The signal is highpass filtered with normalized angular cut-off $\omega_c \approx \pi/4$

(C) The signal is lowpass filtered with cut-off $f_c \approx 33$ Hz

(D) The signal `x` is inverted so that one can listen to the sound backward

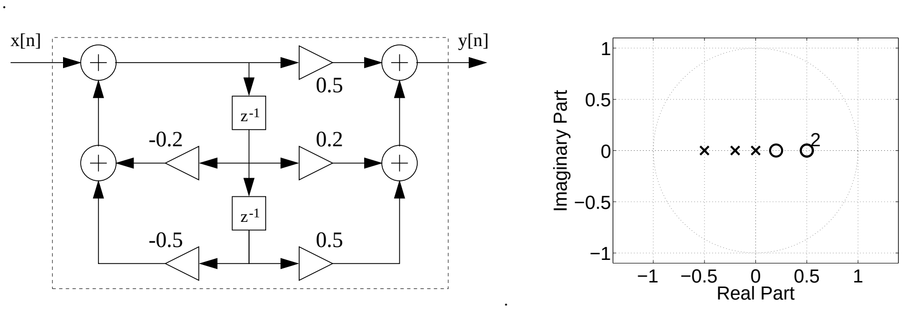

Figure 1: Multichoice statement 1.1: block diagram and pole-zero plot for choice **(C)** .

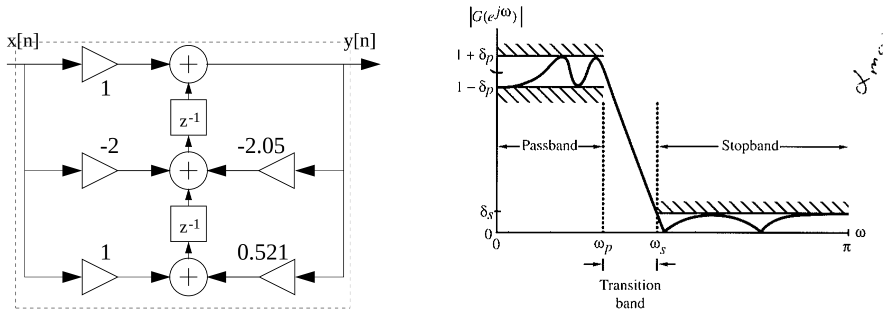

Figure 2: Left, block diagram of Multichoice statement 1.5. Right, specifications of Multichoice statement 1.6.

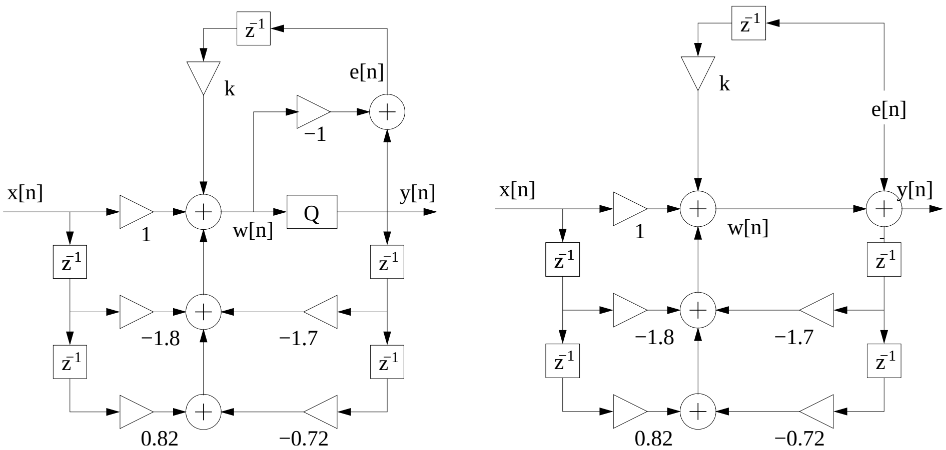

Figure 3: Multichoice statement 1.7 and 1.8, 2nd order IIR with 1st order error-feedback. Right, Q is replaced by noise source $e[n]$.

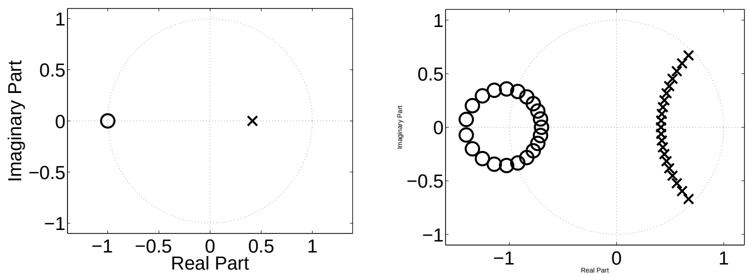

Figure 4: Multichoice statement 1.9: pole-zero plots of 1st order and 21st order lowpass filters.

2) (6p, **MTE2 AND FINAL EXAM**) Choose **either** 2A **or** 2B.

2A) FFT algorithms. In addition to common properties you can also use an example of "radix-2 DIT FFT" algorithm, whose flow diagram is given in Figure 5 for 4 points and $r = 1,\ 2,$ and $l_r = 0, \ldots, 2^{r-1} - 1$. Check out the butterfly equations and W_N from formula paper. Compute transform with intermediate steps for sequence $x[n] = 6\delta[n] + 1\delta[n-1] - 3\delta[n-2] + 1\delta[n-3]$.

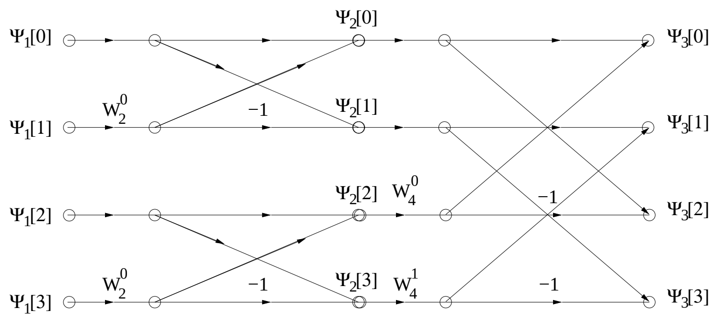

Figure 5: Problem 2A. Flow-diagram of "radix-2 DIT FFT".

2B) We want to have two simple lowpass filters with normalized angular frequency $\omega_c = \pi/3$.

a) Compute 4th order digital filter, whose impulse response is

$$h_{\mathrm{FIR}}[n] = K \cdot \big(a\delta[n] + b\delta[n-1] + c\delta[n-2] + d\delta[n-3] + e\delta[n-4]\big)$$

where parameters a, b, c, d, and e are real numbers and K is constant whose value is not needed here. Explain shortly your method. Give these numbers with two signicant digits, for instance, $\pi \Rightarrow 3.1$.

b) Analog Butterworth lowpass filter with angular frequency Ω_c is of form

$$H(s) = \frac{\Omega_c}{s + \Omega_c}$$

Compute digital 1st order filter

$$H(z) = K \cdot \frac{1 + fz^{-1}}{1 + gz^{-1}}$$

where constant K is not needed. Write down the numbers f and g.

3) (6p, **ONLY FINAL EXAM**)

a) Consider a sequence $x[n] = 2\sin(0.2\pi n + 0.5\pi) - \cos(0.4\pi n + 0.25\pi) - 2\sin(0.5\pi n - \pi/2)$. Compute the fundamental period N_0 of the sequence.

b) A sequence $x_1[n] = x[n]\,\mu[n]$, where $x[n]$ is from (a), is fed into a stable and non-causal LTI system. Explain based on these properties, what do we know about output $y[n]$?

4) (6p, **ONLY FINAL EXAM**) Examine a LTI system, which consists of three subsystems

$$\begin{aligned} h_1[n] &= \delta[n-1] + \delta[n-2] \\ h_2[n] &= \delta[n-1] - 2\delta[n-2] + \delta[n-3] \\ h_3[n] &= \delta[n] - \delta[n-2] + \delta[n-4] - \delta[n-6] \end{aligned}$$

as in Figure 6.

a) Determine the transfer function of the total system $H(z)$

b) If the output of the system is $y[n] = \{1,\ 2,\ \underline{0},\ 2,\ -2,\ 1,\ -2,\ -2,\ 3,\ -3\}$, what has been the input $x[n]$?

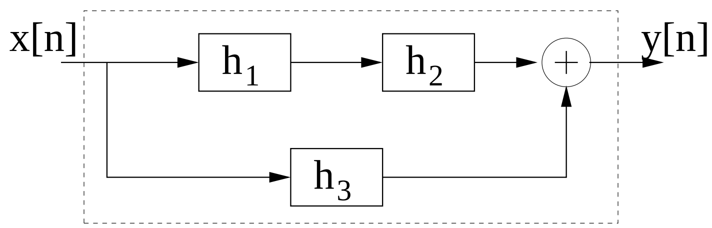

Figure 6: Problem 4. LTI system.

5) (6p, **ONLY FINAL EXAM**) Examine a LTI system in Figure 7.

a) Determine the transfer function $H(z)$ as its simpliest form

$$H(z) = K \cdot \frac{1 + b_1 z^{-1} + b_2 z^{-2} + \ldots}{1 + a_1 z^{-1} + a_2 z^{-2} + \ldots}$$

where constant K is not needed.

b) Draw the block diagram of the filter in canonic (with respect to delays) direct form II structure.

c) Is the filter lowpass / highpass / bandpass / bandstop / allpass? Explain your choice.

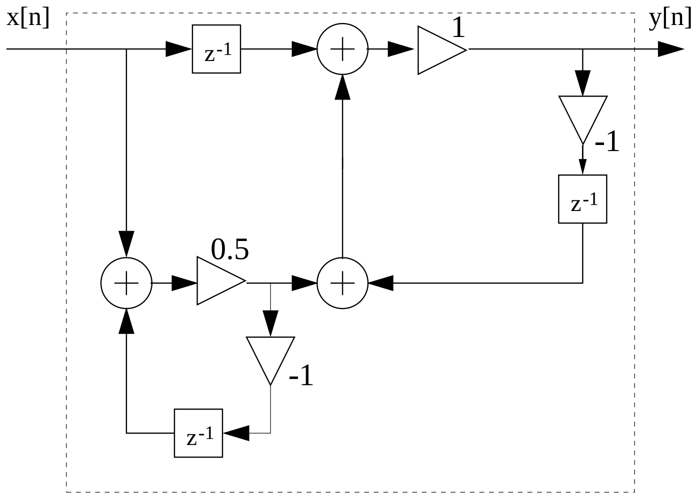

Figure 7: Problem 5. The block diagram of the filter.

6) (6p, **ONLY FINAL EXAM**) Examine stable and causal lowpass filter $H(z)$, whose passband ends at 4 kHz and whose frequency response is 10 kHz. Amplitude response is given in Figure 8(a) and first values of impulse response $h[n]$ in Figure 8(b).

a) Upsample filter with factor $L = 4$. Sketch both the new amplitude response $H^{'}(z) = H(z^4)$ and impulse response $h^{'}[n] = h[n/4]$ for first 10 values.

b) Finally, you'd like to have a lowpass filter with the same cut-off but with 40 kHz sampling frequency. What do you have to do? Sketch both the new amplitude response $|H^{''}(e^{j\omega})|$ and impulse response $h^{''}[n] = h[n/4]$ for first 10 values afterwards.

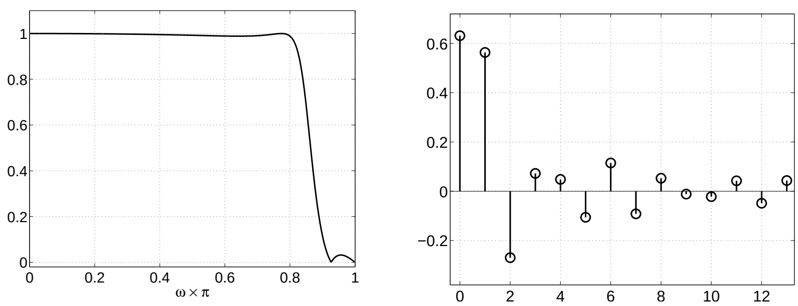

Figure 8: Problem 6. Left, amplitude response. Right, impulse response.