T-61.3010 Digital Signal Processing and Filtering

1st mid term exam, March Tue 6th 2007 at 12-15. Hall M.

You are allowed to take 1st MTE only once either Mar 6th or Mar 10th.

You are not allowed to use any calculators or math books of your own. A formula table and a form for Problem 1 are delivered.

All concept papers have to be returned, **the form from Problem 1 separately.** You can keep the question paper and the formula paper.

Write down all **necessary steps** in your solution of Problem 2.

1) (14 x 1p, max 12 p) Multichoice statements. There are 1-4 correct answers, but choose **one and only one.** Fill in **into a separate form**, which will be read optically.

Correct answer +1 p, incorrect −0.5 p, no answer 0 p. You do not need to explain your choices. Reply to as many statements as you want. The maximum points of this problem is 12 and the minimum 0.

In these statements the region of convergence (ROC) in z-transforms is considered to be the area outside of the outmost pole.

1.1 Impulse response $h[n]$ of LTI system:
(A) the sum $\sum_{n=-\infty}^{\infty} |h[n]|$ converges always
(B) when inserting $\delta[n]$ into a system, we can read output of the system. The output is then $h[n]$
(C) if values $|h[n]| \neq 0$ are bounded (smaller than finite limit B_h), when $n \in -\infty \ldots \infty$, then the system is FIR-filter
(D) is received when computing inverse Fourier-transform of the amplitude response $|H(e^{j\omega})|$

1.2 Consider a sequence $x[n] = x_1[n]+x_2[n]+x_3[n]$, where fundamental periods of subsequences are $N_1 = 4$, $N_2 = 6$, and $N_3 = 8$. What can be said about the fundamental period of $x[n]$?
(A) The fundamental period is $N_0 = 2$
(B) The fundamental period is $N_0 = 24$
(C) The fundamental period is $N_0 = 192$
(D) None of above is true

1.3 Consider a signal which is a sum of two sinusoidals $x(t) = A_1 \cos(2\pi f_1 t) - A_2 \sin(2\pi f_2 t)$. Examine a case, where $A_1 = A_2 \neq 0$, $f_1 = f_2 \neq 0$, and all coefficients are real-valued. Now we can say about $x(t)$:
(A) $x(t) = 0$ for all t
(B) $x(t) = A_3 \cos(2\pi f_3 t + \theta_3)$, where $A_3 \neq 0$, $f_3 = f_1 = f_2$, and θ_3 is constant
(C) $x(t)$ is complex-valued
(D) Spectrum of $x(t)$ is a triangular spectrum whose tail and head are at frequencies $-f_1$ and $+f_1$ Hz, respectively

1.4 Impulse response of a LTI system is $h[n] = \left(\sum_{k=-\infty}^{\infty} \delta[n-6k] + \delta[n-5k]\right) \cdot \mu[5-n]$
(A) Filter is causal
(B) Filter is stable
(C) Filter is FIR
(D) None of above is true

1.5 Linear convolution $z[n] = x[n] \circledast y[n]$ of sequences $x[n] = \{\underline{1},\ 2,\ -3\}$ and $y[n] = \{\underline{0},\ 1,\ -2,\ 1\}$:
(A) $z[n]$ is 6 units long sequence (borders on left the first non-zero term and on right the last non-zero term)
(B) First non-zero element of $z[n]$ is at $n = -1$
(C) $z[n] = \{\underline{0},\ 2,\ 6\}$
(D) $z[n] = \delta[n-1] - 6\delta[n-3] + 8\delta[n-4] - 3\delta[n-5]$

1.6 We know the input $x[n]$ and the output $y[n]$ of the LTI system: $x[n] = 2\delta[n] + 4\delta[n-1] + \delta[n-2]$ and $y[n] = 0.5\delta[n]+1.5\delta[n-1]+1.75\delta[n-2]+1.75\delta[n-3]+1.25\delta[n-4]+0.25\delta[n-5]$.
(A) Filter $h[n]$ is two-point moving average
(B) The order of the filter is 4
(C) The maximum amplification of the amplitude response is 0.25
(D) None of above is true

1.7 In Figure 1 there are four LTI subsystems $h_1[n] = \delta[n]+\delta[n-1]$, $h_2[n] = \delta[n-1]+\delta[n-2]$, $h_3[n] = \delta[n] + \delta[n-2]$, and $h_4[n] = \delta[n-1] - \delta[n-2]$ which form the impulse response $h[n]$ of the total system.
(A) The impulse response of the total system is $h[n] = \delta[n]+3\delta[n-1]+3\delta[n-2]+\delta[n-3]$
(B) The order of the total system is 3
(C) All poles of the total system $H(z)$ are situated in the origo
(D) None of above is true

1.8 There are two peaks in the spectrum of a digital sequence at $\omega = 0.2\pi$ and $\omega = 0.5\pi$ as shown in Figure 2. The sequence is received by sampling a continuous-time signal $x(t)$ with the sampling frequency 1000 Hz. The original $x(t)$ could have been:
(A) $x(t) = \cos(2\pi \cdot 200 \cdot t + \pi/8) + \cos(2\pi \cdot 500 \cdot t - \pi/6)$
(B) $x(t) = 0.01 \cos(2\pi \cdot 100 \cdot t) + 100 \cos(2\pi \cdot 750 \cdot t)$
(C) $x(t) = 10 \cos(2\pi \cdot 1250 \cdot t) + 10 \cos(2\pi \cdot 1600 \cdot t)$
(D) $x(t) = \cos(2\pi \cdot 100 \cdot t - 0.132) - \sin(2\pi \cdot 1250 \cdot t)$

1.9 See the spectrum $|X(j\Omega)|$ of a continuous-time real signal in top row of Figure 3. Sample the signal with sampling frequency $f_s = 10$ kHz. The spectrum of the sequence $x[n]$ in bottom row of Figure 3 is
(A) (a)
(B) (b)
(C) (c)
(D) (d)

1.10 The transfer function is $H(z) = [1 - 0.2z^{-1} + z^{-2}]/[1 - 0.9z^{-2}]$.
(A) The difference equation is $y[n] - 0.2y[n-1] + y[n-2] = x[n] - 0.9x[n-2]$
(B) The order of the filter is four
(C) The impulse response can be computed recursively $h[n] = \delta[n] - 0.2\delta[n-1] + \delta[n-2] + 0.9h[n-2]$
(D) The amplitude response is in Figure 4, where on x-axis there is ω/π

1.11 The impulse response of the filter is $h[n] = (-0.8)^n \mu[n]$ and the output $y[n] = 2 \cdot (-0.8)^n \mu[n] - (-0.4)^n \mu[n]$.
(A) The input is $x[n] = 2\delta[n] - (0.5)^n \mu[n]$
(B) The input is $x[n] = (-0.4)^n \mu[n]$
(C) The filter is averaging the input
(D) The filter is both unstable and non-causal

1.12 Consider a sequence $x_1[n] = \{\underline{2}, 0, 0, 7\}$ whose discrete Fourier transform (DFT) is $X_1[k] = \{\underline{9}, 2+7j, -5, 2-7j\}$. What is DFT $X_2[k]$ of "delayed by two" sequence $x_2[n] = x_1[< n-2 >_4] = \{\underline{0}, 7, 2, 0\}$. Hint: formulas and/or $x[< n - n_0 >_N] \leftrightarrow W_N^{n_0 \cdot k} \cdot X[k]$, where $< . >_N$ is modulo N.

(A) $X_2[k] = \{\underline{-9}, -2-7j, 5, -2+7j\}$

(B) $X_2[k] = \{\underline{9}, -2-7j, -5, -2+7j\}$

(C) $X_2[k] = \{\underline{5}, -2-7j, -9, -2+7j\}$

(D) $X_2[k] = \{\underline{-5}, 2-7j, 9, 2+7j\}$

1.13 What is the discrete-time Fourier-transform $X(e^{j\omega})$ of the sequence $x[n] = \delta[n+3]+\delta[n+2]+\delta[n+1]+\delta[n]+\delta[n-1]$? Hint: formulas.

(A) $X(e^{j\omega}) = e^{-j3\omega} + e^{-j2\omega} + e^{-j\omega} + 1 + e^{j\omega}$

(B) $X(e^{j\omega}) = e^{-j\omega n} \cdot \frac{\cos(2.5\omega)}{\cos(0.5\omega)}$

(C) $X(e^{j\omega}) = e^{j\omega} \cdot \frac{\sin(2.5\omega)}{\sin(0.5\omega)}$

(D) None of above is true

1.14 Let us examine an audio sample $x[n]$, which is read into column vector `x` from the file `nayte.wav` in Matlab. The size of the vector `x` is `44100 x 1`. The sampling frequency is $f_s = 22050$ Hz. The function `zeros` retuns zeros, e.g., `zeros(4,1)` returns a column vector with four zeros.

```
[x, fs] = wavread('nayte.wav');
y1 =       [x; zeros(4410,1)];
y2 = 0.7 * [zeros(2205,1); x; zeros(2205,1)];
y3 = 0.4 * [zeros(4410,1); x];
y  = y1 + y2 + y3;
```

(A) The length of the output $y[n]$ is approximately 0.5 seconds

(B) The output $y[n]$ can be computed as convolution of the input $x[n]$ and the impulse response $h[n]$, where $h[0] = 1$, $h[2205] = 0.7$, $h[4410] = 0.4$, and $h[k] = 0$ elsewhere

(C) The high-frequency noise components have been attenuated in the output $y[n]$

(D) It is possible to create "echo effects" using the corresponding method

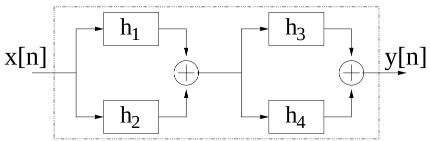

Figure 1: Figure of multichoice Problem 1.7

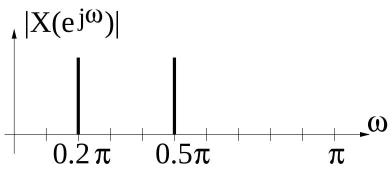

Figure 2: Figure of multichoice Problem 1.8

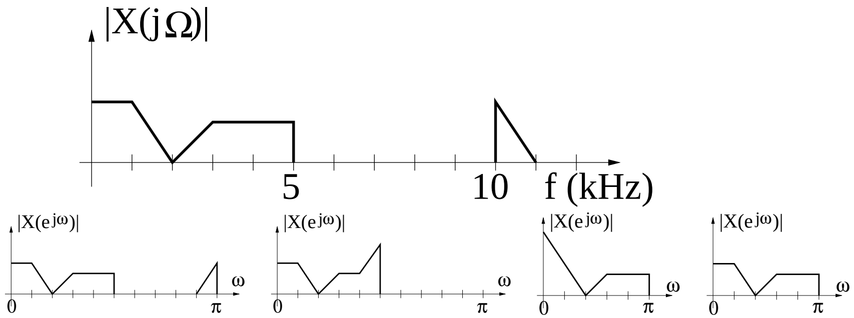

Figure 3: Figures of multichoice Problem 1.9. Top row: continuous $X(j\Omega)$, Bottom row: choices **(A)** , **(B)** , **(C)** , **(D)**

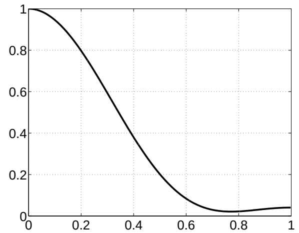

Figure 4: Figure of multichoice Problem 1.10.

2) (6p) Consider a case where you are interviewing a person outside in a noisy environment. The conversation is recorded in a digital format into a laptop computer. There is a narrow-band noise source, which causes distortion in the recording. The quality of the audio is afterwards improved by attenuating frequency components at the frequency of the noise band.

a) What is a suitable sampling frequency when recording speech? What things should the interviewer remember at the recording moment in order to get the part of the noise relatively small compared to the actual speech?

b) Suppose that the noise source has generated pure sine signal with normalized angular frequency ω_r. Filter the signal with the filter

$$H(z) = \frac{(1 - e^{j\omega_r} z^{-1}) \cdot (1 - e^{-j\omega_r} z^{-1})}{(1 - c \cdot e^{j\omega_r} z^{-1}) \cdot (1 - c \cdot e^{-j\omega_r} z^{-1})}$$

where c is chosen in range $0 < c < 1$. Write down the transfer function in the format of

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

where $\omega_r = 3\pi/4$ and $c = 0.9$ and the coefficients b_i and $a_i \in \mathbb{R}$. Draw the pole-zero plot and sketch the amplitude response $|H(e^{j\omega})|$. Hint: formulas "trigonometric functions".

c) How does changing of c in range $0 < c < 1$ affect the filtering properties? Explain.

d) Suppose that the noise is not continuous but randomly occuring, and the frequency changes in (small) range between ω_{r1} and ω_{r2}. Discuss what kind of changes you need to the implementation of (b).