

Tik-61.246 Digital Signal Processing and Filtering

1st mid term exam 28th Oct 2000 at 10-13. Main building/CS-building.

You are allowed to have a mathematical reference book and (graphical) calculator. You are not allowed to save notes in calculator. Follow the instructions by assistant in the hall.

- (2p) Examine if the following sequences are periodic. Calculate the basic period N of periodic sequences.

a) $x_1[n] = 3 \cos(\frac{8}{31}n + \frac{1}{2}\pi)$

b) $x_2[n] = \sum_{k=-\infty}^{\infty} (\delta[n - 1 - 4k] - 2\delta[n + 1 - 4k])$

- (4p) Consider a linear, time-invariant, stable and causal discrete system, where the input $x[n]$ and output $y[n]$ are:

n	$x[n]$	$y[n]$
0	1	2
1	-2	1
2	0	?
3	1	?
4	2	?

- Define the impulse response $h[n]$ of the system using $x[n]$ and $y[n]$, and conditions that initial values of system are zero and it is form (a , b , c and d constants):

$$h[n] = \begin{cases} a, & \text{when } n < 0 \\ b, & \text{when } n = 0 \\ c, & \text{when } n = 1 \\ d, & \text{when } n > 1 \end{cases}$$

- Calculate the missing values of $y[n]$.

(Convolution $y[n] = h[n] * x[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n - k]$)

- (6p) Consider a filter in Figure 1.

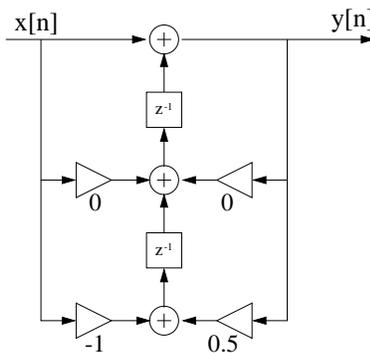


Figure 1: Block structure of Exercise 3

- Define the difference equation and transfer function $H(z)$ of the filter. (Hint: $ax[n - n_0] \leftrightarrow az^{-n_0}X(z)$, $H(z) = Y(z)/X(z)$)
- Calculate and draw the zero-pole-diagram of the filter.

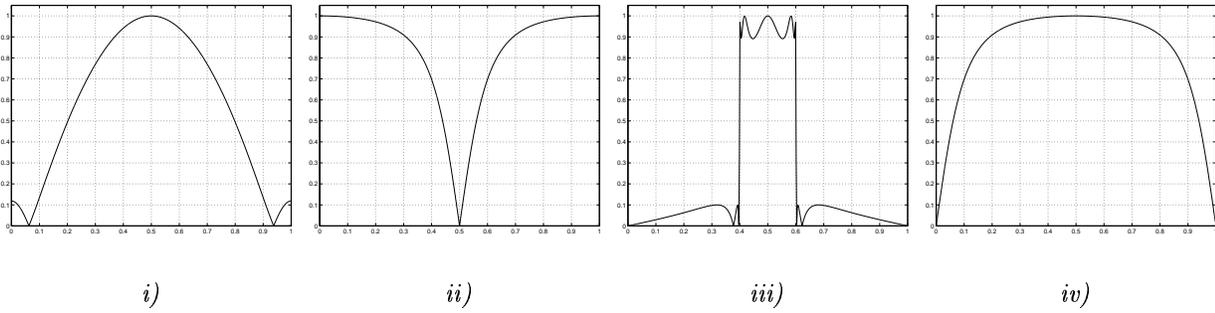


Figure 2: Alternative amplitude responses of Exercise 3c

- c) Choose the most appropriate amplitude response from Figure 2. In figures the frequency axis is $0..f_s/2$ (half of the sampling frequency) and amplitude axis is scaled in $0..1$. What type of filter is it: FIR/IIR, low/high/bandpass/bandstop?
- d) Define $H(z)$ as a combination of two blocks (not unique)

$$H(z) = H_1(z)H_2(z),$$

where $H_1(z)$ and $H_2(z)$ are first order transfer functions (one zero and one pole). Define the transfer functions $H_1(z)$ and $H_2(z)$ and sketch the amplitude responses for both.

4. (6p) Consider a signal

$$x(t) = 3 \cos(2\pi f_1 t) + \cos(2\pi f_2 t) + 2 \cos(2\pi f_3 t),$$

where $f_1 = 4$ kHz, $f_2 = 6$ kHz and $f_3 = 14$ kHz.

- Define the basic period T of signal $x(t)$.
- Sketch the magnitude spectrum $|X(j\omega)|$ of signal $x(t)$ in frequency range $-20 \dots 20$ kHz.
- Sample the signal $x(t)$ using sampling frequency $f_s = 10$ kHz and sketch the magnitude spectrum $|X(e^{j\omega})|$ of discrete sequence $x[n]$ in frequency range $-20 \dots 20$ kHz.
- Use ideal lowpass filter

$$H(j\omega) = \begin{cases} 1, & |f| < 9\text{kHz} \\ 0, & |f| \geq 9\text{kHz} \end{cases}$$

and filter $X_2(j\omega) = H(j\omega)X(j\omega)$. Sketch the magnitude spectrum $|X_2(j\omega)|$ in range $-20 \dots 20$ kHz.

- Sample the filtered signal $x_2(t)$ using sampling frequency $f_s = 10$ kHz and sketch the magnitude spectrum $|X_2(e^{j\omega})|$ of discrete sequence $x_2[n]$ in frequency range $-20 \dots 20$ kHz.