39. Suppose that the calculation of FFT for a one second long sequence, sampled with 44100 Hz, takes 0.1 seconds. Estimate the time needed to compute (a) DFT of a one second long sequence, (b) FFT of a 3-minute sequence, (c) DFT of a 3-minute sequence. The complexities of DFT and FFT can be approximated with $\mathcal{O}(N^2)$ and $\mathcal{O}(N \log_2 N)$, respectively.

40. Express the desimal number -0.3125 as a binary number using sign bit and four bits for the fraction in the format of (a) sign-magnitude, (b) ones' complement, (c) two's complement. What would be the value after truncation, if only three bits are saved.

41. In the following Figure 17, some error probability density functions of the quantization error are depicted.

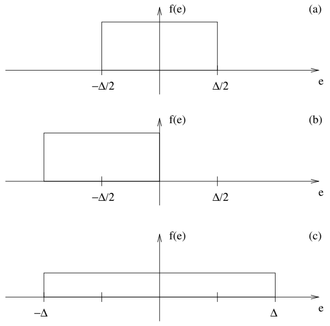

Figure 17: Problem 41, error density functions.

(a) Rounding
(b) Two's complement truncation
(c) Magnitude (one's complement) truncation

is used to truncate the intermediate results. Calculate the expectation value of the quantization error m_e and the variance σ_e^2 in each case.

$\mathrm{E}[E] = \int_{-\infty}^{\infty} f(e)\, e \, \mathrm{d}e$, $\mathrm{Var}[E] = \mathrm{E}[(E - \mathrm{E}[E])^2] = \mathrm{E}[E^2] - (\mathrm{E}[E])^2$

42. In this problem we study the roundoff noise in direct form FIR filters. Consider an FIR filter of length N having the transfer function

$$H(z) = \sum_{k=0}^{N-1} h[k] z^{-k}.$$

Sketch the direct form realization of the transfer function.

a) Derive a formula for the roundoff noise variance when quantization is done before summations.

b) Repeat (a) for the case where quantization is done after summations, i.e. a double precision accumulator is used.

43. The quantization errors produced in digital systems may be compensated by error-shaping filters(Section 9.10 in Mitra). The error components are extracted from the system and processed e.g. using simple digital filters. This way the noise at the output of the system can be reduced.

Consider a lowpass DSP system with a second-order noise reduction system in Figure 18.

a) What is the transfer function of the system if infinite wordlength is used?
b) Derive an expression for the transform of the quantized output, $Y(z)$, in terms of the input transform, $X(z)$, and the quantization error, $E(z)$, and hence show that the error feedback network has no adverse effect on the input signal.
c) Deduce the expression for the error feedback function.
d) What values k_1 and k_2 should have in order to work as an (useful) error-shaping system?

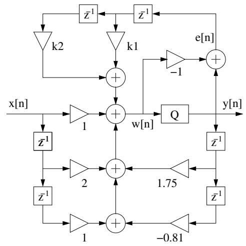

Figure 18: Second-order system with second-order noise reduction in Problem 43.

44. Consider a cosine sequence $x[n] = \cos(2\pi(f/f_s)n)$ where $f = 10$ Hz and $f_s = 100$ Hz as depicted in the top left in Figure 19. While it is a pure cosine, its spectrum is a peak at the frequence $f = 10$ Hz (top middle) or at $\omega = 2\pi f/f_s = 0.2\pi$ (top right).

a) Sketch the output sequence $x_u[n]$ with circles using up-sampler with up-sampling factor $L = 2$, and draw its spectra into second row. Original sequence values of $x[n]$ are marked with crosses. The spectrum in middle column is 0..200 Hz and in right $0..2\pi$, i.e., $0..f_s$.

$$x_u[n] = \begin{cases} x[n/L], & n = 0, \pm L, \pm 2L, \ldots \\ 0, & \text{otherwise} \end{cases} \qquad X_u(e^{j\omega}) = X(e^{j\omega L})$$

b) Sketch the output sequence $x_d[n]$ with circles using down-sampler with down-sampling factor $M = 2$, and draw its spectra into bottom row.

$$x_d[n] = x[nM] \qquad X_d(e^{j\omega}) = \frac{1}{M} \sum_{k=0}^{M-1} X(e^{j(\omega - 2\pi k)/M})$$

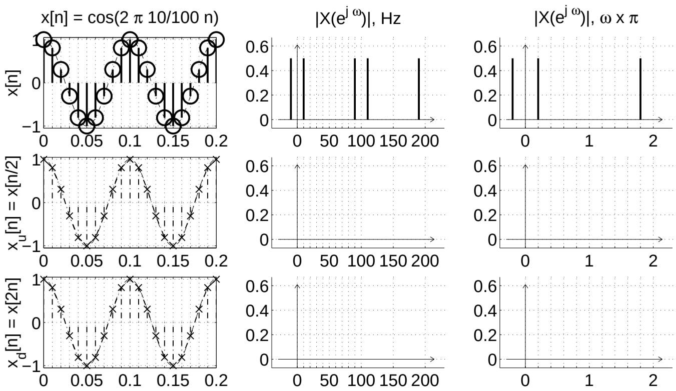

Figure 19: Empty figures for Problem 44. The up-sampling factor $L = 2$, and the down-sampling factor $M = 2$. **Left column**: sequence $x[n]$ with circles, fill in the sequences $x_u[n]$ and $x_d[n]$. X-axis: time (0 . . . 0.2 s). **Middle column**: Spectrum $X(e^{jf})$ (10 Hz component, 100 Hz sampling frequency), fill in the spectra $X_u(e^{jf})$ and $X_d(e^{jf})$. X-axis: frequency (0 . . . 200 Hz). **Right column**: Spectrum $X(e^{j\omega})$ ($2\pi \cdot (10/100) = 0.2\pi$), fill in the spectra $X_u(e^{j\omega})$ and $X_d(e^{j\omega})$. X-axis: angular frequency ($0 \ldots 2\pi$).

45. Express the output $y[n]$ of the system shown in Figure 20 as a function of the input $x[n]$.

46. Show that the factor-of-L up-sampler $x_u[n]$ and the factor-of-M down-sampler $x_d[n]$ defined as in Problem 44 are linear systems.

47. Consider the multirate system shown in Figure 21 where $H_0(z)$, $H_1(z)$, and $H_2(z)$ are ideal lowpass, bandpass, and highpass filters, respectively, with frequency responses shown in Figure 22(a)-(c). Sketch the Fourier transforms of the outputs $y_0[n]$, $y_1[n]$, and $y_2[n]$ if the Fourier transform of the input is as shown in Figure 22(d).

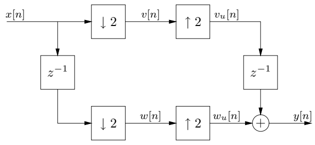

Figure 20: Multirate system of Problem 45.

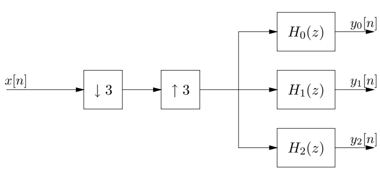

Figure 21: Multirate system of Problem 47.

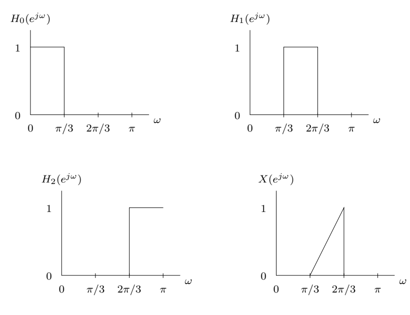

Figure 22: (a)-(c) Ideal filters $H_0(z)$, $H_1(z)$, $H_2(z)$, (d) Fourier transform of the input of Problem 47.