Helsinki University of Technology
Lab of Computer and Information Science

Tik-61.184 Special Course in Information Technology
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Exercises, session 7

1. Assume that the variables s_1 and s_2 have a spherically symmetric joint pdf given by:

$$p(s_1, s_2) = \begin{cases} c, & \text{if } s_1^2 + s_2^2 \leq 1 \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

   a) Compute the constant c.
   b) Show that s_1 and s_2 are uncorrelated in the ordinary linear sense. (Hint: use symmetry)
   c) Show intuitively (verbally, no equations needed) that the correlation of energies is here *negative*, in contrast to the situation discussed in the papers of this session.

2. Assume that the z_i and z_j in equation (6) in the topographic ICA chapter are gaussian, uncorrelated, and of unit variance. a) Write down an integral expression that gives the joint probability density function of s_i and s_j. b) Explain intuitively why the pdf of (s_i, s_j) is spherically symmetric.

3. Give an n-dimensional pdf that is spherically symmetric, and in which the energies have neither positive nor negative correlations. (Hint: the answer is simpler than you think.)