Tik-61.183 Special course on information technology III
Randomized algorithms
Spring 2000
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Exercises 1, due February 9, 2000

1. The variance $D^2(X)$ of a random variable X is the expectation of the random variable $(X - E(X))^2$, i.e., $D^2(X) = E((X - E(X))^2)$. The variance is also denoted by σ_X, or by $\mathbf{var}[X]$. Show that $D^2(X) = E(X^2) - (E(X))^2$.

2. Let $X(n)$ be the number of tails obtained when flipping a fair coin n times. What is $E(X(n))$? What is $D^2(X(n))$?

3. Let $X(n,p)$ be the number of heads obtained when flipping a biased coin (probability of head $= p$) n times. Compute $E(X(n,p))$ and $D^2(X(n,p))$.

4. Find *Markov's inequality* from a textbook and give an informal justification of it. State Chebyshev's inequality.

5. Approximate (either analytically or by simulation) the probability of the following event: "When flipping a fair coin 1000 times, the number of heads is less than 450".