T-61.140 Signal Processing Systems

2nd mid term exam / final exam, Wed 12.5.2004 9-12 C.
You may use a mathematical handbook and graphical calculator. There are formulae on accompanying papers - use them!
2nd mid term exam: Write to the top of the concept “MID TERM EXAM” and **reply to problems 3, 4, 5 and 6.**
Final exam: Write to the top of the concept “FINAL EXAM” and **reply to problems 1, 2, 4, 5, and 6.**

1) (Final exam, 3 x 2p = 6p) Compute or explain clearly.

   a) Discrete-time system is defined by $y[n] = ny[n-1] + 0.5x[n] - 0.25nx[n-1]$. Is the system LTI?

   b) LTI-system is defined by its impulse response $h[n] = \left(\frac{-1}{3}\right)^{n-1} u[n+1]$. Is the system stable? Is the system causal?

   c) Consider a discrete-time sequence $x[n] = 3\cos(\frac{\pi}{6}n) + \sin(\frac{\pi}{4}n - \frac{\pi}{2})$. Is $x[n]$ periodic? If it is, what is the fundamendal period N_0?

2) (Final exam, 6p) Let us examine a cascade LTI system in Figure 1 below. The following impulse responses are known:

$$\begin{aligned} h_1[n] &= \delta[n] + \delta[n-1] \\ h[n] &= \delta[n] + 2\delta[n-1] - \delta[n-2] - 4\delta[n-3] - 2\delta[n-4] \end{aligned}$$

   a) Compute the output of the system $h[n]$ for the input $x[n] = -\delta[n+1] + \delta[n]$.

   b) Compute the impulse response $h_2[n]$. Is $h_2[n]$ causal?

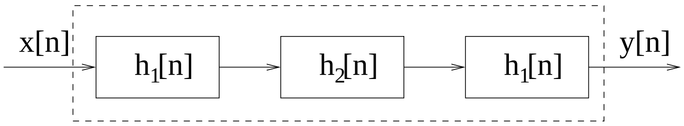


Figure 1: Problem 2, $h[n]$ consists of three LTI subsystems in cascade.

3) (Mid term exam, 3 x 2p = 6p) Reply to **at most three** statements, if it is true (T) or false (F). Explain briefly.

   a) Any sequence $z[n]$ can be recovered from its amplitude spectrum $|Z(e^{j\omega})|$, if there are enough computational power.

   b) If $|H_{BP}(e^{j\omega})|$ is a LTI bandpass filter, whose values are scaled between $0 \ldots 1$, then $|H(e^{j\omega})| = 1 - |H_{BP}(e^{j\omega})|$ is the amplitude response of a bandstop filter.

   c) LTI system $H(e^{j\omega}) = (1/M)\sum_{k=0}^{M-1} e^{-jk\omega}$, where M is a finite integer, has linear phase response.

   d) Second order LTI system, whose difference equation is $y[n] = 0.3y[n-1] + 0.4y[n-2] + 13x[n]$, can be represented with a parallel connection of two first-order LTI-system: $H(e^{j\omega}) = 8/(1 + 0.8e^{-j\omega}) + 5/(1 - 0.5e^{-j\omega})$.

4) (Final exam/Mid term exam, 6p) A discrete-time LTI system is designed so that it for the input

$$x[n] = \left(\frac{1}{5}\right)^n u[n]$$

the output is

$$y[n] = \left(\frac{-4}{5}\right)^n u[n]$$

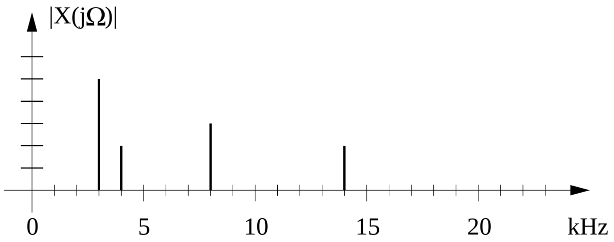
   a) What is the frequency response $H(e^{j\omega}) = Y(e^{j\omega})/X(e^{j\omega})$ of the desired filter?

   b) Sketch the amplitude response $|H(e^{j\omega})|$ in range $0..\pi$. Is the system of type lowpass, highpass, bandpass, bandstop, or allpass?

   c) Write down the corresponding difference equation using $x[n]$ and $y[n]$.

   d) What is the value of the impulse response $h[100]$? Hint: $(-0.8)^{99} \approx -2.54 \cdot 10^{-10}$.

5) (Final exam/Mid term exam, 6p) Consider a real continuous-time signal $x(t)$, which consists of four cosine components. The amplitude spectrum $|X(j\Omega)|$ is shown in the figure below. The highest signal component is 14 kHz. Suppose, that the phase information is zero.



a) Is the signal $x(t)$ periodic? If it is, what is the fundamental frequency?

b) What is smallest sampling frequency, with which there is no aliasing?

c) Signal $x(t)$ is sampled with the sampling frequency $f_s = 12000$ Hz. Sketch the spectrum $|X(e^{j\omega})|$ of the sampled signal $x[n]$ in range $0 \ldots 6000$ Hz.

d) After the sampling the signal is reconstructed back to a continuous-time signal $\hat{x}(t)$. Sketch the spectrum $|\hat{X}(j\Omega)|$ of the signal $\hat{x}(t)$ in range $0 \ldots 23$ kHz.

6) (Final exam/Mid term exam, 6p) **Reply to either A or B.**

6A) Write about speech signal and the analysis of that in the context of this course. How can you preprocess the speech signal so that it is easier to use in speech recognition.

6B) You should produce a sequence of squares $\{0,\ 1,\ 4,\ 9,\ 16,\ 25,\ \ldots\}$ using a third order recursive LTI filter. Figure out or compute the coefficients of the filter and determine the initial values of delay registers. Hint: Think, how a new sample of the sequence can be represented using the previous values. Keep the input always as zero ($x[n] \equiv 0$, for all n).

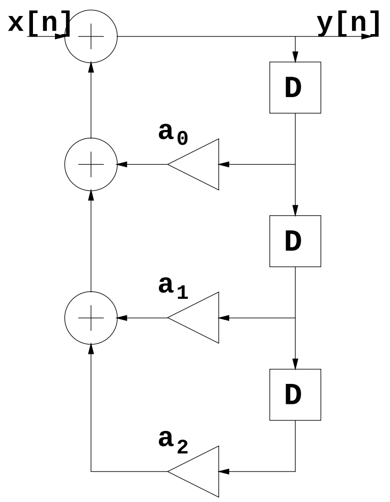


Figure 2: Recursive 3rd order IIR filter.