T-61.140 Signal Processing Systems

2nd mid term exam / final exam, Wed 14th May 2003 9-12 hall C,L
No mathematical reference book. Formulae given - use them! A graphical calculator allowed (extra memory must be cleared).
2nd mid term exam: Write on top “MID TERM EXAM” and **reply to problems 3, 4, 5 and 6.**
Final exam: Write on top “FINAL EXAM” and **reply to problems 1, 2, 4, 5 and 6.**

1) (Final exam, 6p) Consider following continuous-time and discrete-time signals:

$$\begin{aligned} x_1[n] &= \sin(\frac{27}{4}n) \\ x_2(t) &= 2\cos(\frac{27}{4}t + \pi/6) \\ x_3[n] &= \sum_{k=-\infty}^{\infty} \{\delta[n-3k-1] + \delta[n-3k-2]\} \end{aligned}$$

a) Which of the signals are periodic? What is the fundamental period N_0 or T_0 of periodic signals?

b) Determine the fundamental angular frequency ω_0, Fourier-series coefficients and representation for the periodic signals in (a).

2) (Final exam, 6p) Consider a LTI system in Figure 1. It consists of two components, which are connected as shown in (b). The impulse response h_1 of the subsystem is $h_1[n] = \delta[n] - \delta[n-1]$. $h_2[n]$ is unknown. If there is an input $x[n]$ shown in Figure (a), the output $y[n]$ is like in (c).

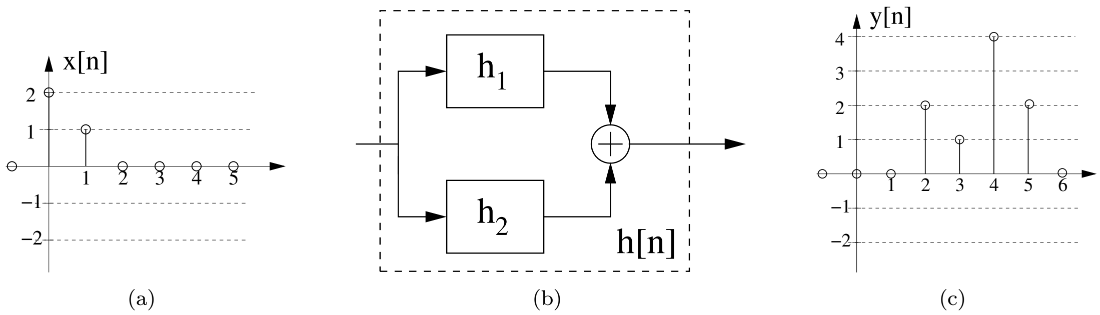


Figure 1: Problem 2: (a) Input $x[n]$, $x[n] = 0$, when $n < 0,\ n > 1$, (b) LTI system, (c) Output $y[n]$, $y[n] = 0$, when $n < 2,\ n > 5,\ n \in Z$.

a) Compute $y_1[n] = h_1[n] * x[n]$.

b) Determine the values $h[0]$ and $h[1]$ of the impulse response.

c) Determine the impulse response $h_2[n]$ of the unknown subsystem..

d) If the input is $x_m[n] = -x[n-1]$, what is the output $y_m[n]$?

3) (Mid term exam, 3 x 2p = 6p) Reply to **three** statements, if they are TRUE (T) or FALSE (F). Explain briefly but unambiguously.

   a) Let us know a real-valued sequence $x[n]$ and its discrete-time Fourier-transform $X(e^{j\omega})$. In frequency $\omega_c = \pi/6 : X(e^{j(\pi/6)}) = (\sqrt{3}/2) + (1/2)j \approx 0.8660 + 0.5j$.
   Statement: $\angle X(e^{j(-\pi/6)}) = -\pi/6$.

   b) The frequency response $H(e^{j\omega}) = \left(-0.2 - e^{-j\omega}\right) / \left(1 + 0.2e^{-j\omega}\right)$ is a highpass filter.

   c) The rise time of a LTI filter $h_1[n] = \sum_{k=0}^{9} \delta[n-k]$ is shorter than that of $h_2[n] = 10 \cdot \sum_{k=0}^{19} \delta[n-k]$. (Rise time is defined in the first page of formulae.)

   d) It is possible to create a lowpass FIR filter so that its group delay is a nonzero constant.

4) (Final exam/Mid term exam, 6p) Consider a discrete-time system, whose block (flow) diagram is in Figure 2.

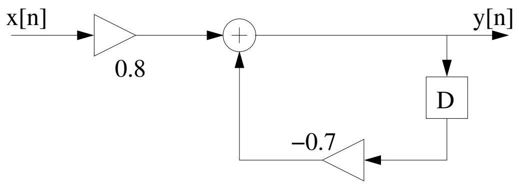


Figure 2: Block diagram in Problem 4.

   a) Is the filter FIR or IIR? Is the algorithm recursive or not? What is the order of the filter?

   b) What is the frequency response $H(e^{j\omega}) = Y(e^{j\omega})/X(e^{j\omega})$.

   c) Sketch the amplitude response $|H(e^{j\omega})|$. Is the filter of type lowpass, highpass, bandpass, bandstop or an allpass filter?

   d) Determine the impulse response $h[n]$ by inverse transform or solving the difference equation.

5) (Final exam/Mid term exam, 6p) Consider an analog real signal which consists of four frequency components of form $\cos(2\pi\, f_k\, t)$). The spectrum of the signal is in Figure 3.

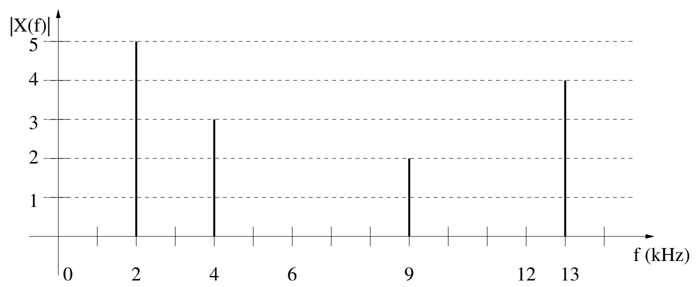


Figure 3: The spectrum of an analog real signal.

   a) Sample the signal with the sampling frequency $f_s = 12$ kHz. What is the time interval T_s? between each sample?

   b) Determine and sketch the discrete-time spectrum $|X(e^{j\omega})|$ in range $0 \ldots 6$ kHz.

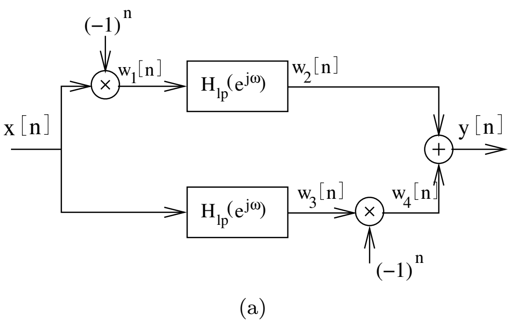
   c) The original analog signal is filtered first with a lowpass filter

$$|H(j\omega)| = \begin{cases} 1, & 0 \le f \le 5 \ \text{kHz} \\ 0.1, & f \ge 6 \ \text{kHz} \end{cases}$$

   The filter has a finite transition band at $5 < f < 6$ kHz. After that the signal is sampled with double sampling frequency $f_s = 24$ kHz. Sketch the discrete-time spectrum $|X_2(e^{j\omega})|$ in $0 \ldots 12$ kHz.

6) (Final exam/Mid term exam, 6p) **Reply to either A or B.**

6A) Consider a discrete-time system shown in Figure 4(a) with input $x[n]$ and output $y[n]$. LTI systems $H_{lp}(e^{j\omega})$ are ideal lowpass filters with cut-off frequency $\pi/4$ and passband amplification of unity. Sketch the output spectrum $|Y(e^{j\omega})|$ in range $(0, \pi)$, when the input is the spectrum $|X(e^{j\omega})|$ of a real-valued sequence $x[n]$ shown in (b). Use the w signals and properties of discrete-time Fourier-transform. Hint: $(-1)^n = e^{j\pi n}$.



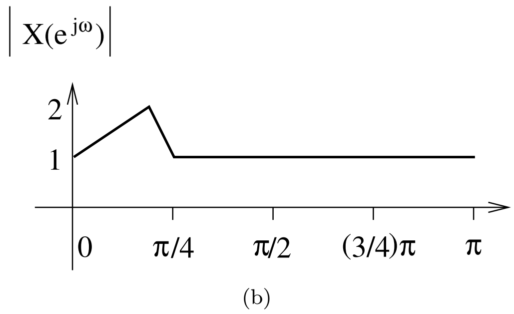
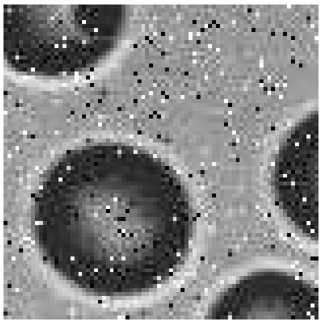


Figure 4: Problem 6A: (a) discrete-time system, (b) the amplitude spectrum $|X(e^{j\omega})|$.

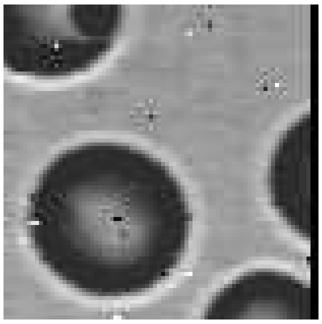
6B) Examine the following code which is written in Matlab (and row numbers are inserted in the beginning of each line). It is applied to 2D-signal, which is represented with graylevel values (range 0-255) in Figure 5(a). Part of the pixels are totally white (255) and some are totally black (0).

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
01: figure, imshow(A,[0 255]);  % plot the original figure (blood cells)
02: number_of_rows = size(A, 1);
03: number_of_cols = size(A, 2);
04: B = zeros(number_of_rows, number_of_cols);  % initialize
05: C = zeros(number_of_rows, number_of_cols);  % initialize
06: for m = 1 : number_of_rows
07:   for n = 1 : number_of_cols-2
08:     temp = A(m, n:n+2);
09:     B(m,n) = mean(temp);
10:     C(m,n) = median(temp);
11:   end;
12: end;
13: figure, imshow(B,[0 255])
14: figure, imshow(C,[0 255])
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Below there are four images. First one (a) is the input signal. Two images of (b-d) are output from the code above while one of them is from another operation. Explain what the program does, how does is relate to two images out of three (b-d), and why these two images look like they do. How does this problem relate to LTI filters in this course?

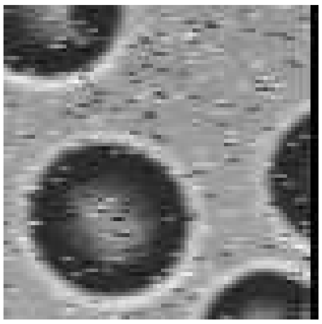


(a)



(b)

(c)



(d)

Figure 5: Images of Problem 6B: (a) original, (b)-(d) filtered images, two out of three images are from the code shown in the text.