Tik-61.140 Signal Processing Systems

2nd mid term exam, Tue 8.5.2001 13-16 D,E,L. (Simula, Koskela, Parviainen)
You may use a mathematical handbook and graphical calculator. There are formulae on accompanying papers - use them!

1. (6p) Answer, if the statement is true (T) or false (F). Correct answer +1p, wrong -1p, no answer 0p
   ATT! There are seven statements, max points 6.

   a) Convolution of input signal and impulse response in time-domain corresponds multiplication of Fourier-transform of input signal and frequency response.
   b) The filter defined by the impulse response $h[n] = \delta[n-1]$ has nonlinear phase.
   c) The smaller the cut-off frequency of a lowpass filter, the longer rise time of step response.
   d) Given amplitude spectrum $|X(e^{j\omega})|$ of any signal $x[n]$, it is possible to recover time-domain sequence $x[n]$ from amplitude spectrum, if there is enought calculation power.
   e) If $|H_{hp}(j\omega)|$ is amplitude spectrum of highpass filter, whose values are scaled between 0..1, then $|H(j\omega)| = 1 - |H_{hp}(j\omega)|$ is amplitude response of lowpass filter.
   f) Ideal lowpass filter is causal.
   g) Coefficients of discrete Fourier-series of discrete-time cosine sequence $x[n] = cos(2\pi f_0 n)$ are $a_{-1} = 0.5$ and $a_1 = 0.5$ while all other a_k, $k = -\infty..+\infty, k \neq \pm 1$, are zero. f_0 is basic frequency.

2. (6p) Suppose we want to design a discrete-time LTI system which has the property that if the input is

$$x[n] = \left(\frac{1}{2}\right)^n u[n]$$

   then the output is

$$y[n] = \left(\frac{1}{5}\right)^n u[n]$$

   a) Find the frequency response of desired system $H(e^{j\omega})$?
   b) Sketch $|H(e^{j\omega})|$ between $0..\pi$. Is the system of type lowpass, highpass, bandpass or bandstop?
   c) What is the impulse response $h[n]$ of the filter?
   d) Find a difference equation relating $x[n]$ and $y[n]$ that characterizes the system.

3. (6p) Suppose that the Fourier-transformed spectrum $X(j\omega)$, in Figure 1, of signal $x(t)$ is known. The highest frequency component is 8 kHz. Suppose that phase information is zero.

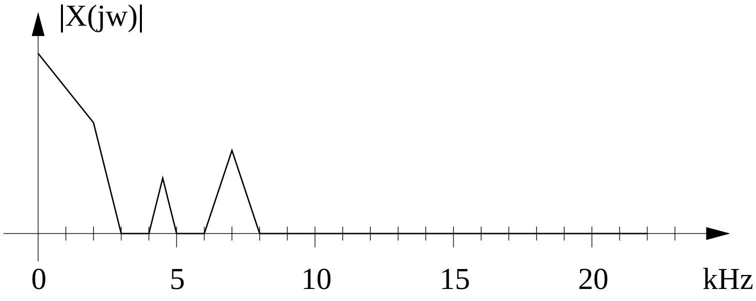

Figure 1: The original spectrum $X(j\omega)$ of Problem 3.

   a) Is the signal $x(t)$ periodic? If it is, what is the basic frequency of it?
   b) Sample $x(t)$ with sampling frequency 22000 Hz. Sketch the sampled specrum $X(e^{j\omega})$ of signal $x[n]$ between 0..11000 Hz. What is the period between samples in microseconds?
   c) Sample $x(t)$ with sampling frequency 10000 Hz. Sketch the sampled specrum $X(e^{j\omega})$ of signal $x[n]$ between 0..5000 Hz.
   d) Explain, how is it possible to decrease aliasing with certain sampling frequency. You can use an ideal lowpass filter, whose cut-off frequency f_c you can choose.

4. (6p) Answer with a few sentences.

   a) What do terms analogue, discrete and digital signal mean?
   b) Why is it useful to process signal digitally?
   c) What errors arise when converting analog signal to digital?
   d) How can you avoid or decrease the errors affected by digitalization?