Tik-61.140 Signal Processing Systems

2. mid term exam, Tue 9.5.2000 16-19 A,B. (Simula, Koskela, Parviainen)
You may use a mathematical handbook and graphical calculator. There are formulae on accompanying papers - use them!

Convolution: $y(t) = h(t) * x(t) = \int h(\tau)x(t-\tau)d\tau,\ y[n] = h[n] * x[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$.

Euler's formula: $e^{j\omega} = cos(\omega) + j\, sin(\omega)$

1. (6p) Answer, if the statement is true (T) or false (F) (a 1 p).

   a) Convolution of signals in time-domain corresponds multiplication of Fourier-transforms in frequency-domain.

   b) The filter defined by the impulse response $h[n] = \delta[n-1]$ has nonlinear phase.

   c) Moving average filter is a lowpass filter of its type.

   d) Spectrum (magnitude of Fourier-transform) of the signal $x[n] = a^n u[n]$ has a peak at frequency $f = \frac{2\pi}{a}$ and elsewhere it is zero.

   e) The inverse Fourier-transform of complex frequency response is not linear.

   f) The parallel connection of a discrete lowpass filter (cut-off frequency $\pi/4$) and a highpass filter (cut-off frequency $3\pi/4$) is a bandpass filter.

2. (6p) Consider a discrete system, which is defined by a difference equation

$$y[n] + \frac{3}{4}\,y[n-1] + \frac{1}{8}\,y[n-2] = \frac{1}{2}\,x[n]$$

   a) Express the frequency response of the system $H(e^{j\omega}) = Y(e^{j\omega})/X(e^{j\omega})$. Hint: use transfer property $x[n-n_0] \leftrightarrow e^{-j\omega n_0}X(e^{j\omega})$.

   b) Calculate some values for $|H(e^{j\omega})|$, for example, when ω gets values $\{0,\ \pi/4,\ \pi/2,\ 3\pi/4,\ \pi\}$. Using these values sketch the amplitude response $|H(e^{j\omega})|$ in the range $0..\pi$. The filter is low, second order, so it behaves smoothly between the points. Hint for quicker calculation by hand: $e^{j\pi/4} \approx 0.7 + 0.7j$.

   c) What is the type of the filter: lowpass, highpass, bandpass, bandstop or allpass (all frequencies multiplied by 1)?

   d) What is the impulse response $h[n]$ of the system?

3. (6p) Consider analog signal, which consists of five frequency components. The spectrum of the signal, where the amplitudes of each frequency components are shown, is in the figure 1. The spectrum is symmetric around the origo - there are only positive frequencies shown.

   Suppose that the phase of each component is zero (signals are of form $cos(2\pi\, f_k\, t)$), so that the impact of phases is not needed to know.

   a) The signal is sampled with the sampling frequency 6 kHz. What is the length of the period between sample points in signal?

   b) Draw the spectrum of the sampled discrete signal in frequency range 0..3 kHz.

   c) The discrete signal of b) was filtered by a filter whose impulse response was $h[n] = \delta[n-3]$. After that the signal was ideally reconstructed back to analog. What differences can be heard between the original analog signal and the reconstructed processed signal?

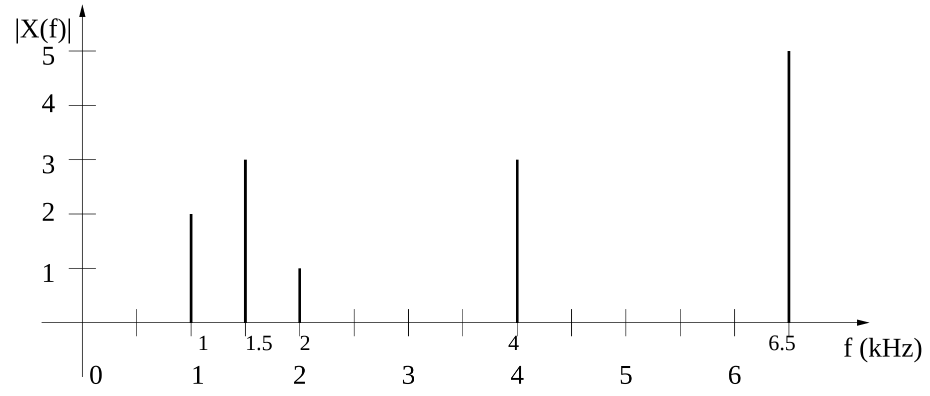

Figure 1: The spectrum of analog signal

d) In problem c) it was noticed that differences were large. In order to decrease the impact of noise and error components the original analog signal was filtered first by a lowpass filter

$$|H(f)| = \begin{cases} 1, & 0 \leq f \leq 2.5\ kHz \\ 0.1, & f \geq 3 \end{cases}$$

The filter has finite transition band at $2.5 < f < 3\ kHz$. Suppose that the filter is zero-phase, so that the phase is not changed. After that the signal is sampled with double sampling frequency, 12 kHz. Draw the spectrum of the new discrete signal at frequencies 0..6 kHz.

4. (6p) There is a fourth-order filter in the figure 2

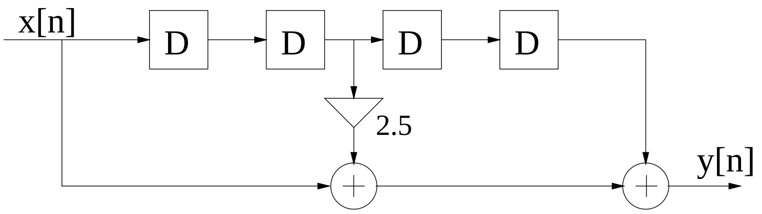

Figure 2: The block diagram of the filter of the problem 4

a) Find the impulse response $h[n]$ from the block diagram.
b) Find the frequency response $H(e^{j\omega})$ of the system.
c) Sketch the amplitude response $|H(e^{j\omega})|$ just like in problem 2b.
d) What is the type of the filter: lowpass, highpass, bandpass, bandstop or allpass (all frequencies multiplied by 1)?
e) When the input is a step function $u[n]$, what is the response of the filter?