Tik-61.140 Signal Processing Systems

1st mid term exam, Mon 12.3.2001 15-18 K,L,N,T1. Simula/Koskela/Parviainen.
You may use a mathematical reference book (for example Beta) and a graphical calculator (clean memory). Some formulae in the additional paper. Write down your answers in concept papers.

1. (4p) LTI systems are by definition linear and time-invariant. A LTI system can be depicted with block diagram, difference equation or impulse response.

   a) Explain briefly, in which way it is possible calculate or show that a given systems is linear and time-invariant.

   b) Which of the following systems $S_1,\ S_2,\ S_3$ are linear and time-invariant? Explain your answers.

$$S_1 : y[n] + 1 = x[n] + x[n-1]$$
$$S_2 : y[n] = |\,x[n]\,|$$
$$S_3 : y(t) - tx(t) = 0$$

   c) Sketch all block diagrams of LTI systems of b).

2. (3p) See Figure 1. There is an analog sum signal of two periodic cosine signals.

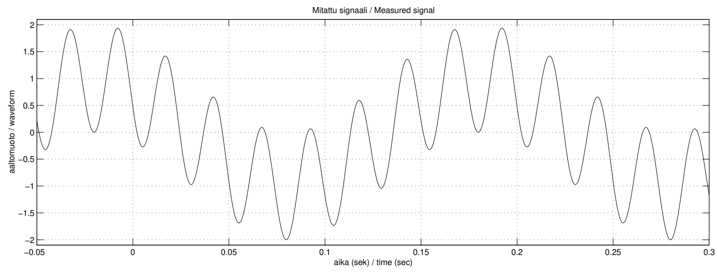

Figure 1: Sum signal

   a) What are the frequencies of two subsignals f_1 and f_2?

   b) What is the basic frequency f_0 of sum signal?

   c) Sketch the two-sided amplitude spectrum (energy of signal as a function of frequency) of sum signal in range -50 Hz .. 50 Hz. Alternatively, you can represent it with Fourier series and basic (angular) frequency, $\omega_0 = 2\,\pi\, f_0$.

3. (6p) Consider a LTI system with impulse response $h[n] = \delta[n] - 2\delta[n-1] + \delta[n-2]$. In addition, it is known that it consists of two similar subsystems $h_1[n]$, which are set in cascade (series).

   a) How long is impulse response $h_1[n]$.

   b) Define the impulse response $h_1[n]$ of the subsystem.

   c) Sketch the block diagram of system $h[n]$.

   d) Calculate the convolution $x[n] * h[n]$, when $x[n]$ is like in Figure 2.

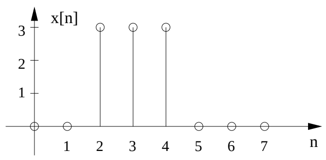

Figure 2: Input x[n]

4. (6p) Consider a continuous-time system whose frequency response is

$$H(j\omega) = \frac{e^{j\omega}}{2 + j\omega}$$

   a) Define the impulse response $h(t)$ of the system.

   b) Suppose that the input is rectangular pulse $x(t) = \begin{cases} 1, & 0 < t < 2 \\ 0, & \text{muualla} \end{cases}$. Define the Fourier transform $Y(j\omega)$ of output.