T-61.140 Signal Processing Systems

2nd mid term exam / final exam, Wed 15.5.2002 9-12 C, L (Simula, Parviainen)
You may use your own graphical calculator. There are formulae on accompanying papers - use them!

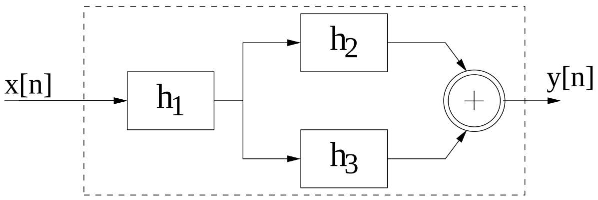
2nd MTE: 3, 4, 5, 6, 7
Exam: 1, 2, 5, 6, 7

Notice! You are not allowed to take part in 2nd MTE, if you attended MTE on 3.5.2002.

1) (6p, EXAM) Properties of signals and systems.

   a) (1p) With which value of α discrete-time system $y[n] = 0.5x[n] + 0.25\alpha x[n-1] + \alpha$ is LTI? Explain.
   b) (1p) Is $y[n] = \sum_{k=-3}^{3} k^2\, x[n-k]$ a stable LTI-system? Explain.
   c) (1p) Is $y[n] = x[-n]$ a causal discrete-time system? Explain.
   d) (1p) Is signal $x(t) = \cos(\frac{2}{9}t)$ periodic? If it is, what is the length of fundamental period T?
   e) (1p) Is sequence $x[n] = \sin(\frac{4}{9}n)$ periodic? If it is, what is the length of fundamental period N?
   f) (1p) What is the fundamental period N of sequence $x[n] = \cos(\frac{\pi}{6}n - \pi) + \sin(\frac{\pi}{9}n)$?

2) (6p, EXAM) LTI system, whose impulse response is $h[n] = -\delta[n-1] + 2\delta[n-2] + 3\delta[n-3]$, consists of three LTI subsystems $h_1[n]$, $h_2[n] = -\delta[n] + 2\delta[n-1] - \delta[n-2]$ and $h_3[n] = \delta[n-1] + \delta[n-2]$, see the picture below.



   a) (1p) Explain, how the impulse response $h_1[n]$ can be derived.
   b) (2p) Compute $h_1[n]$.
   c) (2p) The input is $x[n] = \{\underline{0},\ -1,\ 2\}$. What is the output $y[n]$?
   d) (1p) The system is modified so that the block h_1 is moved to be after h_2 and h_3 (h_2 and h_3 parallel, then h_1 in cascade). The modified system has impulse response $\hat{h}[n]$. What can be said about impulse responses $h[n]$ and $\hat{h}[n]$?

3) (0-3p, 2. VK) Answer, if the statement is true (T) or false (F). Correct answer +1p, wrong -1p, no answer 0p. Explain briefly! **ATT!** There are four statements, max points 3.

   a) Convolution of input signal and impulse response in time-domain corresponds the product of frequency response and Fourier-transform of input signal.
   b) Averaging FIR filter (MA, moving average) is always stable, because it contains a feedback loop.
   c) If the analog signal is band limited, and if the period length of quickest changing component is over two times long as the sampling period, there will not be any aliasing.
   d) The rise time of the filter is defined to be time interval, when unit step response rises from 10% to 90% of its maximum value. Statement: The filter $h[n] = (0.9)^n$ has shorter rise time than $h[n] = (0.1)^n$.

4) (3p, 2nd MTE) Answer with a few sentences.

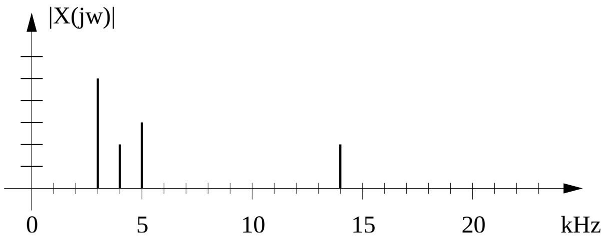
   a) What do terms analog(ue), discrete and digital signal mean?
   b) Why is it useful to process signal digitally?

5) (6p, EXAM and 2nd MTE) LTI system is defined with impulse response

$$h[n] = (-0.8)^n u[n] - (-0.8)^{n-1} u[n-1]$$

   a) (2p) Calculate and draw the values of impulse response, when $n = -2..3$, and sketch the values approximately when $n = 4..10$.
   b) (2p) What is the frequency response $H(e^{j\omega})$ of the system?
   c) (1p) Sketch $|H(e^{j\omega})|$ in range $0..\pi$. Is the system of type lowpass / highpass / bandpass / bandstop?
   d) (1p) Calculate the corresponding difference equation using $x[n]$ and $y[n]$.

6) (6p, EXAM and 2nd MTE) Let us know a continuous-time signal $x(t)$, which consists of four cosine components, and its Fourier spectrum $|X(j\omega)|$, which is depicted in the picture below. The highest signal component is 14 kHz. Let us assume that phase is 0 for each component.



   a) (1p) Is the signal $x(t)$ periodic? If it is, what is the fundamental frequency of that?
   b) (1p) What is the smallest sampling frequency, where no aliasing exists?
   c) (3p) In order to use a digital application the signal $x(t)$ is sampled with $f_s = 12000$ Hz. Sketch the spectrum $|X(e^{j\omega})|$ of the sampled signal $x[n]$ in range 0...6000 Hz.
   d) (1p) The interesting band for the application is 2500...5500 Hz. Explain briefly with a couple of sentences the situation after sampling and the influence of the high-frequency (14 kHz) component.

7) (6p, EXAM and 2nd MTE) **Reply to either A or B.**

7A) LTI system is defined with difference equation

$$y[n] = 0.5\,x[n] + 0.5x[n-1]$$

   a) (1p) Draw the block diagram of LTI system.
   b) (1p) The system computes the mean of two latest samples. Is the system of type lowpass / highpass / bandstop / bandpass?
   c) (2p) Replace each delay with a double delay. Write down a modified difference equation and frequency response $H(e^{j\omega})$?
   d) (2p) Sketch the modified $|H(e^{j\omega})|$ in range $0..\pi$. Is the modified system of type lowpass / highpass / bandstop / bandpass?

7B) Write down an essay of frequency-selective LTI-filters and filtering on the basis of the course. The maximum length of the essay is two pages with line space of two; illustrations can be drawn to clarify the text.