Bayesian Theory reading group
Exercise problems for Chapter 6

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1. Present examples related to your own work of model comparison/selection problems in $\mathcal{M}$-closed, $\mathcal{M}$-completed and $\mathcal{M}$-open settings.

2. Let us assume we have N samples of data from

$$p(\mathbf{x}) = \mathcal{N}\left(\mathbf{x};\ \mathbf{0}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}\right), \quad \rho \in (-1, 1).$$

   We have two alternative Gaussian models with a *diagonal* covariance, $M_1 : p_1(\mathbf{x}) = \mathcal{N}(\mathbf{x};\ \mathbf{0}, \mathbf{\Sigma}_1)$ and $M_2 : p_2(\mathbf{x}) = \mathcal{N}(\mathbf{x};\ \mathbf{0}, \mathbf{\Sigma}_2)$. The covariance matrices are selected so that the Kullback–Leibler divergences $D_{KL}(p||p_1)$ and $D_{KL}(p_2||p)$ are minimal, i.e.

$$\mathbf{\Sigma}_1 = \mathbf{I}, \quad \mathbf{\Sigma}_2 = (1 - \rho^2)\mathbf{I}.$$

   The prior probabilities are $p(M_1) = p(M_2) = \frac{1}{2}$.

   (a) Assume (incorrectly) an $\mathcal{M}$-closed setting. What is the optimal model for the data

      i. for point predictions of future observations with respect to quadratic loss?

      ii. with respect to a proper local score function for the predictive distribution?

   (b) Assume an $\mathcal{M}$-completed setting. What is now the optimal model for the data

      i. for point predictions of future observations with respect to quadratic loss?

      ii. with respect to a proper local score function for the predictive distribution?