Mining music graphs through immanantal polynomials

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Abstract

Graphs represent an effective tool for modeling structural features of music in symbolic format. Here we show how it is possible to characterize a music graph by means of the second immanantal polynomial and how to embed the polynomial coefficients into a low dimensional vector space, biased on specific music collections, by means of Independent Component Analysis, thus allowing for music mining through the standard inner product.

1. Introduction

A new model oriented to mining of similarly structured musical themes has been proposed in (Pinto & Haus, 2007) relying on graph theory. The underlying idea is that melodic sequences can be clustered by partitioning them into equivalence classes characterized by having isomorphic representative graphs. The next step is mining similar graphs by means of a similarity function which makes use of graph powers. Graphs are labeled and the similarity function involves all possible isometries and graph powers up to a fixed degree. This makes the model not very suitable for online applications. In this paper our approach is different and relies basically on a characterization of graphs with less than 12 nodes by their second immanantal polynomial.

The rest of the paper is organized as follows. First we introduce the graph model and present the main results about algebraic graph theory for the characterization of melodies in section 2. In section 3 we present a clustering method based on the embedding of invariant vectors in a three dimensional space through independent component analysis (ICA) and finally in section 4 we provide an evaluation of this technique against the *Themefinder* database of classical themes.

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2. Graph model

Let M be a theme of length $m = |M|$ and consider the sequence of pitches $\{h_s\}_{s\in\mathbb{I}}$, $\{\mathbb{I} = 1, ..., m\}$. Let $V = \mathbb{Z}_{12}$ be the metric space of pitches, or pitch classes, in the 12-tone system and consider the linear graph G_l obtained by associating a vertex labelled by h_s to every element $h_s \in V$ and an edge $a_s : h_s \to h_{s+1}$ to every couple (h_s, h_{s+1}), so that $\partial_0 a_s = h_s$ and $\partial_1 a_s = h_{s+1}$ (Bollobás, 1998). The music graph which represents the melodic sequence is obtained by quotienting the vertex set by identifying the vertex with the same label (Haus & Pinto, 2005).

In order to endow the set of representative graphs (one for each melody) with a suitable concept of distance a spectral approach has been proposed in (Pinto et al., 2007). One way to characterize a graph is in fact to use the spectral properties of some matrixes related to it. The concept of music similarity is independent from node permutation, so the graph distance measure has to be invariant under permutations of vertexes and this is the main reason to deal with spectra of those matrices (Cvetkovi et al., 1995), (Sarkar & Boyer, 1996) and (Umeyama, 1988).

The problem with graph spectra is that they do not really characterize a graph as two non isomorphic graph can share the same spectrum. (Zhu & Wilson, 2005) provides an estimation of the percentage of cospectral graphs with 11 vertices both for the adjacency matrix (∼21%) and for the laplacian matrix (∼9%). In other words, we are looking for a polynomial $p(G)$ associated with a graph G such that

$$\begin{cases} \text{if } G_1 \neq G_2 & \text{then} \quad p(G_1) \neq p(G_2) \\ \text{if } p(G_1) \neq p(G_2) & \text{then} \quad G_1 \neq G_2 \end{cases} \tag{1}$$

As for the equals in equations 1, we remind that for graphs equal means isomorphic and for polynomials it means to have the same degree and coefficients. As graph isomorphism is known as an NP-complete problem, in general it is not easy to establish without doubt if two graphs are isomorphic or not. There are many invariants and necessary conditions one can exploit but by no means they can provide criteria for graph isomorphism.

Nevertheless, for graphs whose order is less than twelve it is known that the *second immanantal polynomial* $d_2(xI - L(G))$ of the laplacian matrix has the property 1 and its coefficients are computable in polynomial time (n^4) (Cvetkovi et al., 1995) (Sossa et al., 1992).

Such a property does not hold for graphs whose size is greater than 12 but for music this is not (and for the standard western notation not at all) a strong limitation, as music graphs relative to standard western music notation have at most 12 nodes. The d_2 polynomial associated with the laplacian matrix $L(G)$ is:

$$\begin{aligned} d_2(xI - L(G)) &= c_0(G)x^n - c_1(G)x^{n-1} + \dots \\ & \quad +(-1)^n c_n(G) \end{aligned} \tag{2}$$

The coefficients $c_0, \dots, c_n$ can be computed as follows:

$$\begin{aligned} \mathbf{c_0}(G) &= n-1 \\ \mathbf{c_1}(G) &= 2m(n-1) \\ \mathbf{c_k}(G) &= \sum_{X \in Q_{k,n}} \Big(\sum_{i=1}^{n} l_{i,i} \det \big(L(G)\{X\}(i)\big) \\ & \quad - \det \big(L(G)\{X\}\big) \Big) \end{aligned} \tag{3}$$

$Q_{k,n}$ denotes the collection of C_n^k k-element subsets of the set $\{1, 2, \dots n\}$. If we denote with $L[X]$ the $k \times k$ principal submatrix of M corresponding to X, where $X \in Q_{k,n}$, we can define the $m \times m$ matrix

$$L\{X\} = \begin{pmatrix} L[X] & 0_k \\ 0_k & I_{n-k} \end{pmatrix} \tag{4}$$

where I_{n-k} is the identity matrix of size $n-k$, 0_k is the null matrix of size k and $L\{X\}(i)$ is the matrix obtained from $L\{X\}$ by removing the n-th row and the n-th column.

3. Graph embedding

At this point we are able to partition the set of representative graphs into equivalence classes with the equivalence relation provided by sharing the same second immanantal polynomial, that is to say the same characteristic vector $c = (c_0, c_1, \dots, c_n)$.

In this section we are going to introduce a technique to get a better clustering of the characteristic set in order to provide a more reliable metric for the representative graphs, specifically designed for the graphs actually present in the database. In fact, our objective is to provide a metric that allows to make queries on a large collection of representative graphs, or better of their characteristic vectors c. As they come from a music database, they are supposed not to be an arbitrary distribution of twelve-dimensional vectors, thus it is a good rule to find out the main characteristics, or components, of the specific collection.

Here this task is worked out by an Independent Component Analysis (ICA) algorithm which has been shown to perform best in image classification tasks (Luo et al., 2003). The idea is to find the most independent components, in the interval $(0, 12)$, of all the characteristic set and then to project each vector on these independent components (Hyvarinen & Oja, 2000).

We used the first three most significant independent components $\mathbf{e^1}, \mathbf{e^2}, \mathbf{e^3}$ to represent the characteristic vectors extracted from the themes. The coordinate system is spanned by the the three independent components $\mathbf{e^1}, \mathbf{e^2}, \mathbf{e^3}$ and the characteristic vectors $\mathbf{c^j}$ can be projected onto this pattern space by $x_i^j = \mathbf{e^i} \cdot \mathbf{c^j}$, where $i = 1, 2, 3,\ j = 1, 2, \dots, N$ and N is the dimension of the database. It follows immediately that the metric will be the standard one between vectors in this linear space.

4. Experimentation

The experiments have been carried out on musical incipits taken from the *Themefinder* database developed at the Center for Computer Assisted Research in the Humanities of Stanford University. First of all the *Themefinder* format (differential) is converted into a note sequence. Then adjacency and laplacian matrixes are computed for each graph, together with the d_2-coefficients. Afterwards an independent component analysis algorithm is applied to the set of coefficients in order to extract the main three independent components.

In Figure 1 three themes of the database randomly chosen have been windowed and embedded in the clustering space all windowed themes in order to investigate how different "views" of the same object can be embedded. The third picture in Figure 1 represents the embedding of windowing of the third randomly chosen theme; in the second picture we added also the cluster of the windowing process applied to the second randomly chosen theme. The first picture includes all three clusters. From these plots is it possible to draw some conclusions. Clusters are evident in Figure 1 and so it is possible to partition the scattered plot into equivalence classes corresponding to the different orbits. We can also remark that it is possible that there are intersections between different orbits, as in the first picture of Figure 1 where there is an element of the first orbit which is quite close to the orbit of the second theme. This is because the original themes may share a similar structure but only in some windows.

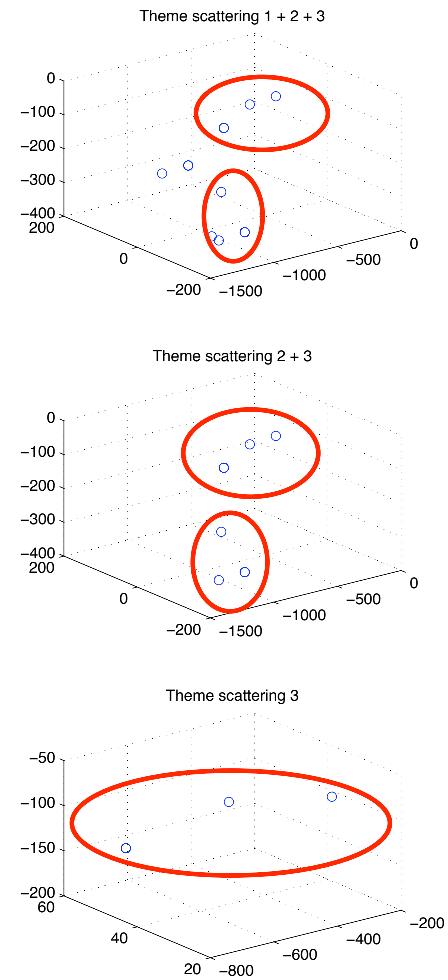

Figure 1. Orbit clusters for variations of 3 themes. The third plot represents only the third set of variations, the second plot the second and the third sets and in the first plot all variations are present.

5. Conclusions

We presented a music graph characterization through the second immanantal polynomial, which provides an invariant set of coefficients which completely characterizes a music graph. This allows for music graph partitioning in polynomial time. Then it is possible to embed the set of polynomial coefficients into a low dimensional space (a three dimensional space) where it is possible to give a graph metric tuned on the specific collection by means of the standard product. This space is spanned by the independent vectors provided by the Independent Component Analysis performed on all vectors belonging to the database.

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